Estimation of the double-helicity-flip deuteron structure function

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The gluonic hadron structure function $\Delta(x,Q^2)$, identified by Jaffe and Manohar, is estimated for the deuteron. We assume a model for the deuteron which contains a Δ-Δ isobar component, and in which the virtual photon interacts with each isobar independently. Although the predicted transverse asymmetry cross section is small, the results could be tested in view of a proposed experiment to measure $\Delta(x,Q^2)$.

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It is of fundamental interest in hadron physics to measure the contribution of gluons in deep-inelastic scattering processes. This is not easily done because the virtual photon has no direct coupling with gluons, and must perforce interact through a mechanism such as the box diagram. The contribution of this must be added onto the usual tree-level γ^*-q diagram, and this makes isolation of the gluon component very difficult. Fortunately, as pointed out by Jaffe and Manohar [1], favorable experimental circumstances can sometimes be arranged in which the gluons alone contribute. More precisely, one needs a polarized hadron target with spin $J\geq 1$, and the signature for the gluonic contribution must be the presence of a double-helicity-flip amplitude. In the language of the operator-product expansion (OPE), there exists a tower of gluon operators $G_{\mu\mu_1}\overleftrightarrow{D}_{\mu_3}\cdots\overleftrightarrow{D}_{\mu_n}G_{\nu\mu_2}$ that contributes for spin $J\geq 1$ and gives rise to the double-helicity-flip structure function $\Delta(x,Q^2)$. The coefficient functions of these operators, of order $\alpha_s(Q^2)$, are obtained from box graphs. Anomalous dimensions of these operators have also been calculated recently [2]. In addition to $\Delta(x,Q^2)$, there also exist other interesting (but quark-dominated) structure functions for $J\geq 1$ [3]. It is expected that a proposed experiment at the DESY *ep* collider HERA, using an internal gas-jet polarized deuteron target, will be able to measure these various structure functions, including $\Delta(x,Q^2)$ [4].

We first quickly recall some basic formulas and definitions [1] for $J=1,\frac{3}{2}$. In the Bjorken limit, the structure function $\Delta(x,Q^2)$ is defined by $\Delta=\varepsilon^{*\mu}(-)W_{\mu\nu}^{+-}\varepsilon^{\nu}(+)$, where $\varepsilon_\mu^{(\lambda)}$ is the polarization vector of the virtual photon and $W_{\mu\nu}^{+-}=(1/2\pi)\mathrm{Im}T_{\mu\nu}^{+-}$. Here $T_{\mu\nu}^{+-}$ is the amplitude for forward scattering of the incident virtual photon corresponding to a change of 2 units of angular momentum of the hadron target, with the spin quantization axis being along $\hat{\mathbf{z}}$. Fortunately, Δ has a direct parton-level interpretation. Imagine that the hadron ($J\geq 1$) is placed in an infinite-momentum frame moving in the $\hat{\mathbf{z}}$ direction, the hadron spin being aligned in the $\hat{\mathbf{x}}$ direction. Define $g_x(x,Q^2),g_y(x,Q^2)$ to be the probability of finding a gluon with momentum fraction x having linear polarization along the $\hat{\mathbf{x}},\hat{\mathbf{y}}$ directions, respectively, and denote their difference by

$$a(x,Q^2)=g_{\hat{x}}(x,Q^2)-g_{\hat{y}}(x,Q^2)\ . \tag{1}$$

Then, in terms of $a(x,Q^2)$, the $\Delta(x,Q^2)$ structure function is

$$\Delta(x,Q^2)=\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}\mathbb{Q}^2x^2\int_x^1\frac{dy}{y^3}a(y,Q^2)\ , \tag{2}$$

where $\mathbb{Q}$ is the charge matrix diag$(\frac{2}{3},-\frac{1}{3},-\frac{1}{3})$. Experimentally, $\Delta(x,Q^2)$ is measurable through the ϕ dependence of the cross section, $d\sigma\propto\Delta(x,Q^2)\cos2\phi$. Here ϕ is the azimuthal angle between the plane formed by the beam and the alignment axis, and the plane formed by the beam and the scattered electron. For further details we refer the reader to the original work [1] on $\Delta(x,Q^2)$.

DEUTERON PROBABILITY

Consider now the scattering of leptons off a deuteron. If the deuteron is merely a bound state of two nucleons then $\Delta(x,Q^2)$ is identically zero; it is impossible for a spin-$\frac{1}{2}$ object to flip its spin by 2 units.[1] However, the transient existence of $J\geq 1$ nucleon resonances can change this conclusion. Working in this conventional baryon picture, the deuteron state can be expressed as

$$|D\rangle=|NN\rangle+\varepsilon|\Delta\Delta\rangle+\cdots\ . \tag{3}$$

As an isospin-zero object the deuteron has no Δ-N component; the Δ's must come in pairs. Estimates for ε^2, the total Δ-Δ component, range from 0.01 to 0.03 [5–8]. Spectator experiments have sought to measure this component by knocking out one Δ and detecting the other by its subsequent decay into a nucleon and pion. The validity of the two-step model illustrated in Fig. 1 is assumed; i.e., the incident virtual photon interacts with only one of the Δ's. The experiment is totally inclusive. However, we parenthetically observe that if the observation of the

[1]Scattering from two nucleons, i.e., final-state interactions, could flip the deuteron spin by 2 units and hence contribute to Δ. However, this is a higher-twist effect which vanishes in the Bjorken limit.

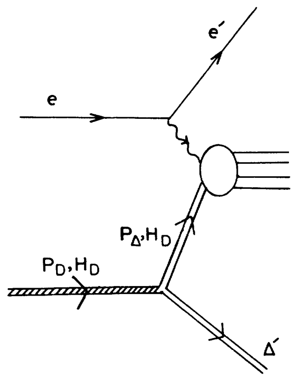

FIG. 1. Scattering of an electron off the Δ-Δ component of a deuteron target.

spectator Δ was experimentally possible, it would be very interesting to study the isospin dependence of the struck Δ's $\Delta(x,Q^2)$.

Under the assumption that probabilities can be convoluted we have

$$g_i^{H_D}(x)=\int dy\,dz\,\delta(x-yz)\sum_{H_\Delta} h_{H_\Delta}^{H_D}(y)g_i^{H_\Delta}(z)\quad (i=\hat{\mathbf{x}},\hat{\mathbf{y}})$$
$$\equiv \sum_{H_\Delta} h_{H_\Delta}^{H_D}\otimes g_i^{H_\Delta}\ . \tag{4}$$

In the above $g_i^D(x)$ is the probability of finding a gluon of linear polarization i carrying fraction x of the deuteron's momentum when the deuteron has spin H_D in the $\hat{\mathbf{x}}$ direction, $H_D=\pm 1,0$. The quantity $g_i^{H_\Delta}(z)$ has a corresponding meaning. $h_{H_\Delta}^{H_D}(y)$ is the probability of finding the struck Δ with momentum fraction y for given H_D and H_Δ.

It follows from parity and time-reversal invariance that

$$a_D^{H_D=1}=a_D^{H_D=-1}=-\tfrac{1}{2}a_D^{H_D=0}\equiv a_D \tag{5}$$

and

$$a_\Delta^{H_\Delta=3/2}=a_\Delta^{H_\Delta=-3/2}=-a_\Delta^{H_\Delta=1/2}=-a_\Delta^{H_\Delta=-1/2}\equiv a_\Delta\ , \tag{6}$$

where $a_D^{H_D}=g_{\hat{x}}^{H_D}-g_{\hat{y}}^{H_D}$ and $a_\Delta^{H_\Delta}=g_{\hat{x}}^{H_\Delta}-g_{\hat{y}}^{H_\Delta}$. Inserting these relations into (4), and using $h_{H_\Delta}^{H_D}=h_{-H_\Delta}^{-H_D}$ yields

$$a_D(x)=\Delta h\otimes a_\Delta\ ,$$
$$\Delta h(y)=h_{3/2}^{1}(y)+h_{-3/2}^{1}(y)-h_{1/2}^{1}(y)-h_{-1/2}^{1}(y)\ . \tag{7}$$

We first concentrate on estimating $\Delta h(y)$. For this, one must have as input the momentum-space distribution of Δ's in a deuteron. Since a relativistic treatment is not available, we shall have to make do with the nonrelativistic wave functions of Refs. [5–8] wherein

$$\psi_{\Delta\Delta}^{H_D}(\mathbf{P}_\Delta)=\sum_{LS}u_{LS}(\mathbf{P}_\Delta)[Y_L(\hat{\mathbf{P}}_\Delta)\chi_S]_{J_D=1,H_D}\ . \tag{8}$$

Symmetry considerations limit the values of (L,S) to (0,1), (2,1), (2,3), and (4,3). The radial wave functions are obtained [5–8] as approximate solutions to the Schrödinger equation with $g_{N\Delta\pi}$ taken from the known $\Gamma_{\Delta\to N\pi}$ decay rate. The infinite-momentum-frame (IMF) momentum distribution $h_{H_\Delta}^{H_D}(y)$ is obtained from the laboratory-frame distribution $h_{H_\Delta}^{H_D}(\mathbf{P}_\Delta)=|\langle \mathbf{P}_\Delta H_\Delta | H_D\rangle|^2$ according to the convolution model prescription:

$$h_{H_\Delta}^{H_D}(y)=\int d^3P_\Delta\,\delta\left[y-\frac{P_\Delta^+}{P_D^+}\right]h_{H_\Delta}^{H_D}(\mathbf{P}_\Delta)\left[1+\frac{P_\Delta^3}{M_\Delta}\right]\ , \tag{9}$$

where $P^+=P^0+P^3$. Performing the operation of projecting the isobar spin from Eq. (8), and then summing over all m quantum numbers, a convenient form for $\Delta h(y)$ can be written down:

$$\Delta h(y)=\sum_{L'S'LSl}Y(L'S'LSl)\int d^3P_\Delta\,\delta\left[y-\frac{P_\Delta^+}{P_D^+}\right]u_{L'S'}^*(P_\Delta)u_{LS}(P_\Delta)Y_{l0}(\hat{\mathbf{P}}_\Delta)\ , \tag{10}$$

where Y is composed of standard Wigner 3-j's, Racah W coefficients, and 9-j symbols according to

$$Y(L'S'LSl)=\left[\frac{5}{\pi}\right]^{1/2}[J_D][L']^{1/2}[L]^{1/2}[S']^{1/2}[S]^{1/2}[l]^{1/2}(-1)^{L+S'}$$
$$\times\begin{pmatrix}L' & L & l\\ 0 & 0 & 0\end{pmatrix}W(\tfrac{3}{2}S',\tfrac{3}{2}S,\tfrac{3}{2}2)\sum_{\mathcal{J}}[\mathcal{J}]\begin{pmatrix}1 & 1 & \mathcal{J}\\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix}\mathcal{J} & 2 & l\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}l & \mathcal{J} & 2\\ L & 1 & S\\ L' & 1 & S'\end{Bmatrix}\ . \tag{11}$$

In the above, $[J]=2J+1$.

Equation (10) is incomplete unless the energy of the Δ is specified inside the deuteron, which is at rest in the laboratory ($P_D^+=M_D$). We assume nonrelativistic kinematics: $P_\Delta^0=M_\Delta+P_\Delta^2/2M_\Delta-\varepsilon$ with ε about equal to the Δ-N mass difference. Clearly the Δ is far off mass shell, and this treatment is valid only as a first estimate. We also note that, having chosen the spin quantization axis along $\hat{x}$ in the argument of the δ function, $P_\Delta^3=P_\Delta\sin\theta\sin\phi$. This implies that the integration over angles must yield a form different from that which would be obtained in the more usual case of longitudinal polarization for which $P_\Delta^3=P_\Delta\cos\theta$. The numerical evaluation of $\Delta h(y)$ is shown in Fig. 2. The wave functions

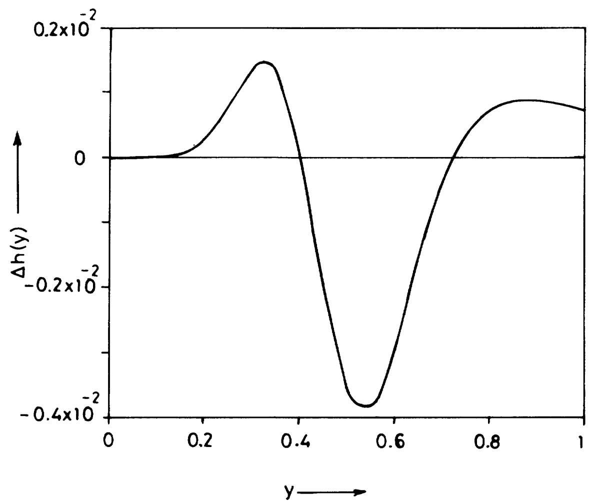

FIG. 2. The difference of the deuteron's transversely polarized Δ probabilities $\Delta h(y)$, as defined in Eq. (7).

$u_{LS}(P_\Delta)$ have been taken from Nath and Weber [9] and are, perhaps, overly conservative in so far as the total Δ-Δ probability is only $\varepsilon^2 \approx 0.009$. One would, therefore, reasonably expect that the scale of $\Delta h(y)$ could be larger by a factor of 2–3.

STRUCTURE FUNCTION

Let us now turn towards calculation of $\Delta_\Delta(z,Q^2)$ or, equivalently, $a_\Delta(z,Q^2)$, since they are related through Eq. (2). The first moment of Δ_Δ has been calculated by Sather and Schmidt [2]; we seek here a generalization to the structure function itself. From Ref. [1], use of the operator-product expansion yields the nth moment

$$\int_0^1 dz\, z^{n-1}\Delta_\Delta(z) = \frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}Q^2 \frac{A_n}{n+2} \ . \tag{12}$$

The "reduced matrix element" A_n is the expectation value of a tower of gluon operators in the isobar rest-frame state polarized along $\hat{x}$ with $H_\Delta = \frac{3}{2}$:

$$A_n = \left[\frac{i}{P_\Delta^+}\right]^{n-2} \langle H_\Delta = \tfrac{3}{2} | G^a_{x+}\partial_+^{n-2}G^a_{x+} - G^a_{y+}\partial_+^{n-2}G^a_{y+} | H_\Delta = \tfrac{3}{2}\rangle \ . \tag{13}$$

Here, $G^a_{x+} = -\partial_+ A^a_x$, etc., because in the light-cone gauge $A^a_+ = 0$. For the $n=2$ case this yields, as in Ref. [1],

$$\int_0^1 dz\, z\Delta_\Delta(z) = \frac{\alpha_s}{8\pi}\mathrm{Tr}Q^2 \frac{1}{P_\Delta^{+2}} \langle H_\Delta = \tfrac{3}{2} | (E^a_y - B^a_x)^2 - (E^a_x + B^a_y)^2 | H_\Delta = \tfrac{3}{2}\rangle \ . \tag{14}$$

Consider now the quantity $a_\Delta(z)$, related to A_n by a Mellin transform:

$$A_n = \int_0^1 dz\, z^{n-1} a_\Delta(z) \ . \tag{15}$$

We seek a form for $a_\Delta(z)$ that yields Eq. (13). Motivated by the usual form encountered in quark distributions, let us guess that

$$\begin{aligned} z a_\Delta(z) &= \frac{1}{P_\Delta^+}\int \frac{d\xi^-}{2\pi} e^{izP_\Delta^+\xi^-} \langle G^a_{y+}(\xi^-)G^a_{y+}(0) + G^a_{y+}(0)G^a_{y+}(\xi^-)\rangle - (x \rightleftharpoons y) \\ &= -\frac{1}{2P_\Delta^+}\int \frac{d\xi^-}{2\pi} e^{izP_\Delta^+\xi^-} \langle G^a_{L+}(\xi^-)G^a_{L+}(0) + G^a_{L+}(0)G^a_{L+}(\xi^-)\rangle + (L \rightleftharpoons R) \ , \end{aligned} \tag{16}$$

where $G_{L+} = G_{x+} - iG_{y-}$ and $G_{R+} = G_{x+} + iG_{y+}$. The field-strength tensor G is related to the color electric and magnetic fields $G_{x+} = -(E_x + B_y)$ and $G_{y+} = -E_y + B_x$. Inserting Eq. (16) into Eq. (15) and performing some simple manipulations confirms the correctness of the guess. For $a_\Delta(z)$ to have a probabilistic interpretation it is necessary that it vanish for $|z| > 1$. Also, $a_\Delta(z) = -a_\Delta(-z)$. Both properties are readily established from the above.

Evaluation of the matrix element indicated in Eq. (16) requires a definite model for the hadronic state. The MIT bag model [10,11] is perhaps adequate, at least as a first estimate. A Δ, aligned in the $\hat{x}$ direction with $H_\Delta = \frac{3}{2}$, is represented by three massless quarks all in the lowest cavity mode. We need not worry about the color-electric field in Eq. (16) because $\mathbf{E}^a = \sum_{i=1}^3 \mathbf{E}^a(i)$ and this, being proportional to $\sum_{i=1}^3 \lambda^a(i)$, annihilates the color-singlet hadron state. The color magnetic field produced by the color current of s-state quarks is obtained from the solution $\nabla \times \mathbf{B}^a = \mathbf{j}^a$, subject to the bag boundary conditions. Explicitly, the solution is [12]

$$\mathbf{B}^a(\mathbf{r}) = g\frac{\lambda^a}{2}[\boldsymbol{\sigma} f(r) + \hat{\mathbf{r}}(\boldsymbol{\sigma}\cdot\hat{\mathbf{r}})h(r)] \ , \tag{17}$$

where $f(r)$ and $h(r)$ are certain elementary functions [12]. Before using this, one must take into account that the expectation value in Eq. (16) must be taken in a state normalized according to

$$\langle P_\Delta H_\Delta | P_\Delta H_\Delta \rangle = (2\pi)^3 2P_\Delta^0 \delta^3(0) \ , \tag{18}$$

whereas a bag state is normalized to unity. With these remarks we directly write down the expression for $a_\Delta(z)$:

$$za_\Delta(z) = \frac{2}{M_\Delta}\int d^3k\,\delta\left[z - \frac{k^+}{P_\Delta^+}\right]_{\text{bag}} \langle H_\Delta = \tfrac{3}{2}|B_x^{a2}(\mathbf{k}) - B_y^{a2}(\mathbf{k})|H_\Delta = \tfrac{3}{2}\rangle_{\text{bag}} . \tag{19}$$

In the above $\mathbf{B}^a(\mathbf{k})$ is the Fourier-transformed version of Eq. (14):

$$\mathbf{B}^a(\mathbf{k}) = g\frac{\lambda^a}{2}[\sigma f(k) + \hat{\mathbf{k}}(\sigma\cdot\hat{\mathbf{k}})h(k)] , \tag{20}$$

with

$$f(k) = \left[\frac{2}{\pi}\right]^{1/2} \int_0^R dr\, r^2\{j_0(kr)f(r) + \tfrac{1}{3}h(r)[j_0(kr) + j_2(kr)]\} , \tag{21}$$

$$h(k) = -\left[\frac{2}{\pi}\right]^{1/2} \int_0^R dr\, r^2 j_2(kr)h(r) . \tag{22}$$

Since $k^+ = k^0 + k\cos\theta$, it is necessary to specify the gluon energy k^0 which occurs in Eq. (19). The B field in Eq. (17) has been obtained semiclassically, but in terms of first-order perturbation theory it corresponds to the field of a $l=1$ transverse electric (TE) gluon in the lowest-cavity mode. Taking the expectation value of the various combinations of σ operators, and performing the angular integration in Eq. (19), we arrive at the final expression for $a_\Delta(z)$:

$$za_\Delta(z) = -64\pi g^2 \times \int_{k_{\min}}^\infty dk\,k\left[f(k) - \tfrac{1}{2}h(k)\left[1 - \frac{k_{\min}^2}{k^2}\right]\right] , \tag{23}$$

where $k_{\min} = |M_\Delta z - k^0|$.

NUMERICAL ESTIMATION

Numerical estimation of $a_\Delta(z)$ in Eq. (23) requires that various bag parameters be specified. The Δ-N splitting determines $\alpha_s = g^2/4\pi \approx 2.2$, $\omega R = 2.04$ for s quarks, and $k^0R = 2.75$ for the lowest-energy gluon mode. The bag radius is $R = 1.1$ fm. Numerical results for $za_\Delta(z)$ and $z\Delta_\Delta(z)$ are plotted in Fig. 3. The first moment of $\Delta_\Delta(z)$ is $-0.012\alpha_s$, which is identical to that found in Ref. [2].

We may now combine a_Δ and Δh according to Eq. (7) to arrive at the deuteron distribution a_D. This is inputted into Eq. (2), and the results for $xa_D(x)$ and $x\Delta_D(x)$ are plotted in Fig. 4. By way of comparison, it is interesting to ask what the isobar contribution to the usual, spin-averaged structure function $F_1^D(x)$ would look like—if a spectator Δ was detected in coincidence, this would be $F_{1\Delta}^D(x)$ itself. This is plotted in Fig. 5, and the ratio $\Delta_D(x)/F_{1\Delta}^D(x)$ is shown in Fig. 6. There are two independent reasons why $\Delta_D(x)$ is small: first, the small probability of finding an isobar in the deuteron, and second, the smallness of $\Delta_\Delta(z)$ itself. The first can be dealt with to an extent by using a heavier nucleus, but the second is inescapable. Detecting the gluonic component of nuclei therefore seems to require an increase in precision of measuring deep-inelastic-scattering (DIS) cross sections by 2–3 orders of magnitude. The proposed experiment at HERA [4] is expected to place only broad constraints on the size of $\Delta_\Delta(x)$.

Finally, we remark upon the validity of our calculations. First, we have assumed a convolution approach. This is quite well justified for weakly bound nucleons in a nucleus, although whether or not it explains European Muon Collaboration type effects is still open to question. However, with Δ's it may not be equally justified because these are much more strongly bound (and thus quite relativistic). Whether bound Δ's and free Δ's have substantially the same structure functions is also open to question. Second, we have relied upon the MIT bag model to calculate $\Delta_\Delta(x)$. But, as is well known, bag models have

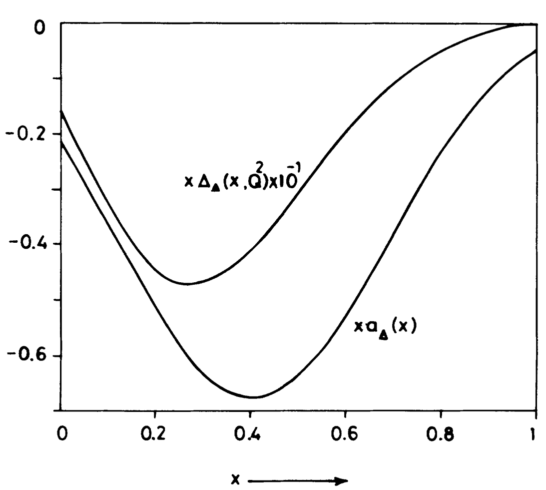

FIG. 3. $xa_\Delta(x)$ and $x\Delta_\Delta(x,Q^2)$ for the Δ in the bag model.

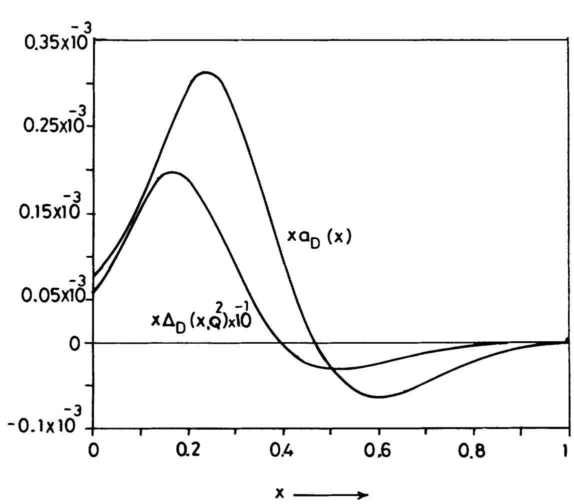

FIG. 4. $xa_D(x)$ and $x\Delta_D(x,Q^2)$ for a deuteron.

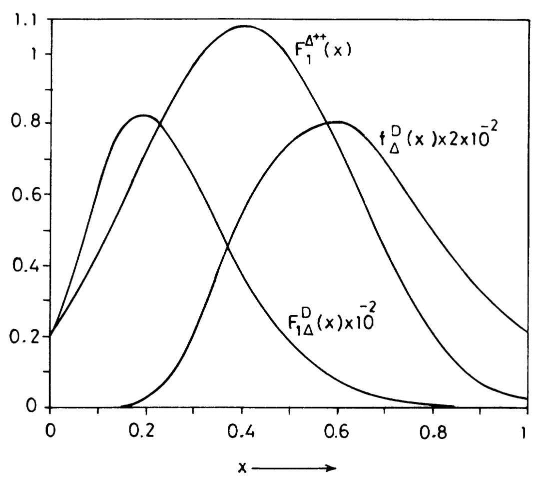

FIG. 5. $F_1^{\Delta^{++}}(x)$ for a bag model Δ^{++} and $f_\Delta^D(x)$, the spin-averaged probability of finding a Δ with momentum fraction x in a deuteron. Also plotted is $F_{1\Delta}^D(x)$, the Δ component contribution to $F_1(x)$ of the deuteron.

definite limitations of accuracy when used to calculate structure functions. This is reflected by nonvanishing support outside of the interval (0,1), and can be seen in our results as well (Fig. 3). While improved methods are available, for an exploratory calculation the treatment in this paper may be adequate.

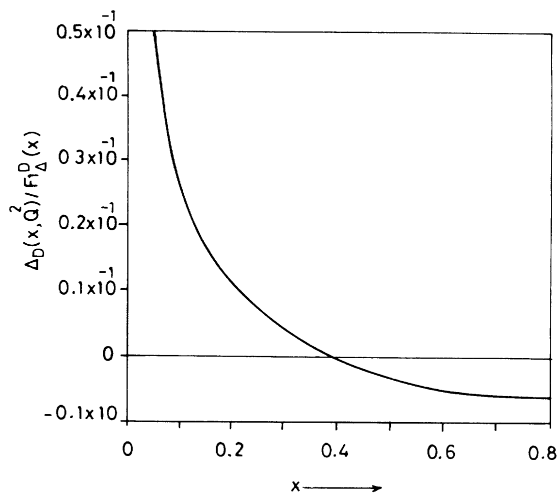

FIG. 6. The calculated ratio of $\Delta_D(x,Q^2)$ to $F_{1\Delta}^D(x)$.

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