Implications of color gauge symmetry for nucleon spin structure

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We study the chromodynamical gauge symmetry in relation to the internal spin structure of the nucleon. We show that (1) even in the helicity eigenstates the gauge-dependent spin and orbital angular momentum operators do not have gauge-independent matrix element, (2) the evolution equations for the gluon spin take very different forms in the Feynman and axial gauges, but yield the same leading behavior in the asymptotic limit, and (3) the complete evolution of the gauge-dependent orbital angular momenta appears intractable in the light-cone gauge. We define a new gluon orbital angular momentum distribution $L_g(x)$ which *is* an experimental observable and has a simple scale evolution. However, its physical interpretation makes sense only in the light-cone gauge just like the gluon helicity distribution $\Delta g(x)$. [S0556-2821(99)02207-9]

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The spin structure of the nucleon has been a subject of intense debate for about ten years. Much progress has been made on both experimental and theoretical frontiers [1]. However, some of the fundamental theoretical issues remain unsettled, as exemplified by a number of recent works in the literature. In particular, in analyzing the spin structure of the nucleon, color gauge invariance is still the cause of some confusion. In this paper, we intend to explore several important and relevant issues in detail.

To set the stage, let us briefly recall the forms of angular momentum operator in quantum chromodynamics (QCD). In Ref. [2], a natural gauge-invariant expression is introduced

$$\vec{J}_{\rm QCD}=\vec{J}_q+\vec{J}_g\,, \tag{1}$$

where

$$\vec{J}_q=\int d^3x\left[\psi^\dagger\frac{\vec{\Sigma}}{2}\psi+\psi^\dagger\vec{x}\times i\vec{D}\psi\right],$$

$$\vec{J}_g=\int d^3x\vec{x}\times(\vec{E}\times\vec{B}). \tag{2}$$

The quark contribution $\vec{J}_q$ contains two terms. The first term is obviously the quark spin as $\vec{\Sigma}=\mathrm{diag}(\vec{\sigma},\vec{\sigma})$ is the four-dimensional generalization of the familiar Pauli spin matrices. The second term is the quark *kinetic* (or mechanical) orbital angular momentum, in which the covariant derivative $\vec{D}=-\vec{\nabla}+ig\vec{A}$ originates from the quark kinetic momentum [3]. We recall that in a gauge theory the kinetic momentum appears to be more physical than the dynamical (or canonical) one, the latter corresponding to a partial derivative $-i\vec{\nabla}$ in quantum mechanics [3]. The gluon contribution to the angular momentum $\vec{J}_g$ contains the well-known Poynting vector, $\vec{E}\times\vec{B}$, the momentum density of the radiation field. Some recent studies in terms of the above form of angular momentum operators can be found in Refs. [4–6].

The same QCD angular momentum can be written in an "interaction-independent" form[1]

$$\vec{J}=\int d^3x\left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi+\psi^\dagger\vec{x}\times(-i\vec{\nabla})\psi+\vec{E}\times\vec{A}+E_i(\vec{x}\times\vec{\nabla})A_i\right]. \tag{3}$$

Because some terms contain explicitly partial derivatives and gauge potentials, the above expression is not *manifestly* gauge invariant. Nonetheless, the physical meaning of each term appears to be clear. The first term is the quark spin, the second the dynamical (canonical) quark orbital angular momentum, the third the gluon "spin," and the last term the gluon "orbital" angular momentum.

According to the above decomposition, one can write down a sum rule for the nucleon spin [7]

$$\frac{1}{2}=\frac{1}{2}\Delta\Sigma(Q^2)+L_q'(Q^2)+\Delta g(Q^2)+L_g'(Q^2). \tag{4}$$

Here the matrix elements of the individual operators are defined in a nucleon state with $p^\mu=(E,0,0,p)$ and helicity 1/2, e.g.,

$$\Delta g(Q^2)=\left\langle p^\mu\frac{1}{2}\right|\int d^3\vec{x}(\vec{E}\times\vec{A})^z\left|p^\mu\frac{1}{2}\right\rangle. \tag{5}$$

The Q^2 dependence results from the renormalization of the composite operators. One expects that L_q', Δg and L_g' are gauge as well as frame dependent. The purpose of this paper is to study how the gauge dependence affects the physical

[1]Of course, it is not really interaction-free because the color electric field still depends on the coupling constant g,

$$\vec{E}^a=-\vec{\nabla}A^{0a}-\frac{\partial\vec{A}}{\partial t}-gf^{abc}\vec{A}^bA^{0c}.$$

significance of the individual terms in the above sum rule. In the following we will work in the infinite momentum frame in which the angular momentum operators are defined from the angular momentum density $\int d^3xM^{+ij}$. In particular, the color electric field E^i is now F^{i+}.

In a recent paper by Chen and Wang [8], it was claimed that although the individual operators in Eq. (3) are gauge invariant, they have gauge-independent matrix elements in the nucleon helicity eigenstates. In other words, L_q', Δg, and L_g' were said to be gauge invariant. If correct, the theorem would have warranted a fresh look at the physical significance of L_q', Δg, and L_g'.

We find that the theorem seems to be in contradiction with the following explicit calculation. Consider an ''on-shell'' quark in the state of momentum p^μ and helicity 1/2. We calculate the matrix element of the gluon spin operator $S_g^z = \int d^3x(\vec{E}\times\vec{A})^z$ in perturbation theory. Choosing the light-cone $A^+=0$, we find at one-loop level

$$\Delta g = \frac{3}{2}C_F\frac{\alpha_s}{2\pi}\ln\left(\frac{Q^2}{\mu^2}\right), \tag{6}$$

where $C_F=(N_c^2-1)/(2N_c)$ with N_c the number of colors, Q^2 and μ^2 are the ultraviolet and infrared cutoffs, respectively. On the other hand, in the covariant gauge we have

$$\Delta g = C_F\frac{\alpha_s}{2\pi}\ln\left(\frac{Q^2}{\mu^2}\right). \tag{7}$$

A similar discrepancy was found upon calculating the matrix element of the same operator in an ''on-shell'' gluon state. Although Chen and Wang's statement of the theorem and proof have been modified from its original version in light of the above example [8], we still doubt the validity of the claim.

Thus the concept of the gluon spin contribution to the nucleon spin is in general a gauge-dependent one. This feature is also reflected in the scale evolution of Δg. In the light-cone gauge $A^+=0$, it is well-known that Δg evolves according to the Altarelli-Parisi equation [9],

$$\frac{d\Delta g(Q^2)}{d\ln Q^2}y = \frac{\alpha_s}{2\pi}\left(\frac{3}{2}C_F\Delta\Sigma + \frac{\beta_0}{2}\Delta g(Q^2)\right), \tag{8}$$

where $\beta_0 = 11-2n_f/3$ with n_f the number of active quark flavors. In the asymptotic limit $Q^2\to\infty$, the gluon spin grows logarithmically,

$$\Delta g(Q^2)|_{\text{axial gauge}} \to \ln Q^2, \tag{9}$$

where the coefficient of proportionality is fixed by nonperturbative physics.

In the Feynman gauge, the evolution equation becomes much more complicated. In fact, the following gauge-variant operators come to mix with the gluon spin

$$O_1 = -\int d^3x\vec{\nabla}A_a^+\times\vec{A}_a, \tag{10}$$

$$O_2 = -\int d^3x g f^{abc}A^{+c}\vec{A}^b\times\vec{A}^a. \tag{11}$$

(There is no ghost operator here because the ghosts do not carry spin.) Denote the matrix elements of the above operators in the nucleon helicity states as a_1 and a_2. A lengthy calculation yields the following evolution equation:

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta g\\ a_1\\ a_2\\ \Delta\Sigma\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix}\frac{7}{8}C_F-\frac{n_f}{3} & \frac{5}{12}C_A & \frac{1}{12}C_A & C_F\\ \frac{23}{24}C_A & \frac{17}{12}C_A-\frac{n_f}{3} & -\frac{1}{12}C_A & \frac{1}{2}C_F\\ -\frac{3}{2}C_A & -\frac{3}{2}C_A & \frac{17}{24}C_A-\frac{n_f}{3} & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\Delta g\\ a_1\\ a_2\\ \Delta\Sigma\end{pmatrix}, \tag{12}$$

where $C_A=N_c$. Thus to evolve the gluon spin to a new perturbative scale, one needs not only the gluon spin at the starting scale but also the matrix elements of O_1 and O_2. To find out the asymptotic behavior as $Q^2\to\infty$, we diagonalize the upper 3×3 mixing matrix. The three eigenvalues are $\lambda_1=(11/6)C_A-n_f/3=\beta_0/2$, $\lambda_2=(17/24)C_A-n_f/3$, and finally $\lambda_3=(11/24)C_A-n_f/3$. From these, we found out that the leading asymptotic behavior of the gluon spin in the Feynman gauge is the same as that in the light-cone gauge,

$$\Delta g(Q^2)|_{\text{Feynman gauge}} \to \ln Q^2. \tag{13}$$

Of course, the coefficients of proportionality in the two gauges are different.

Given that the gluon spin is a gauge-dependent concept, it is remarkable that its value in the light-cone gauge can be extracted from the gluon polarization distribution measurable in high-energy scattering. What one extracts in those experiments is of course gauge-invariant and is in fact the matrix element of the following gauge-invariant non-local operator [10]:

$$O_g = \int_{-\infty}^{\infty} dx \frac{n^-}{2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} F^{+\alpha}(\lambda n) e^{-ig\int_0^\lambda dyn\cdot A(yn)} \tilde{F}_\alpha^{+}(0). \tag{14}$$

However, the physical interpretation of this operator is in general not obvious. Interestingly, in the light-cone gauge $A^+ = 0$, the above operator reduces to the gluon spin operator S_g^z. This relationship says nothing about the gauge transformation property of the gluon spin; it merely means that the gluon spin in the axial gauge can be obtained from the matrix element of a gauge-invariant operator. In other words, the *gauge-invariant extension* of the gluon spin in light-cone gauge can be measured. (This situation is similar in spirit to the following example of length in special relativity. The proper length of a pencil is clearly frame independent. When we say the length of a house in the frame $v = 0.9999c$ is the same as the proper length of the pencil, we are not saying that the length of the house is frame-independent. Rather, we are saying that the length of the house in a special frame can be known from measuring a frame-independent quantity.) Note that one can easily find gauge-invariant extensions of the gluon spin in other gauges. But we may not always find an experimental observable which reduces to the gluon spin in these gauges. As far as the nucleon spin structure is concerned, however, the gluon spin in the covariant gauge is as interesting as its counterpart in the light-cone gauge.

Finally, we turn to the orbital angular momentum operators in Eq. (3). The role of the orbital angular momentum in parton splitting processes was first studied by Ratcliffe [11]. In [12], Tang and two of us worked out the leading-logarithmic scale dependence of the orbital angular momenta in the light-cone gauge,

$$\frac{d}{d\ln Q^2}\begin{pmatrix} L_q' \\ L_g' \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\begin{pmatrix} -\frac{4}{3}C_F & \frac{n_f}{3} \\ \frac{4}{3}C_F & -\frac{n_f}{3} \end{pmatrix}\begin{pmatrix} L_q' \\ L_g' \end{pmatrix} + \frac{\alpha_s(Q^2)}{2\pi}\begin{pmatrix} -\frac{2}{3}C_F & \frac{n_f}{3} \\ -\frac{5}{6}C_F & -\frac{11}{2} \end{pmatrix}\begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}. \tag{15}$$

The first term on the right-hand side exhibits the effects of self-generation of the orbital angular momenta. The second represents the generation of orbital angular momenta from the quark and gluon spin. The above equation leads to some interesting results about the spin structure of the nucleon in the asymptotic limit. As we are going to show below, however, the actual operator mixing is more complicated than what is shown in the above equations although the result in the asymptotic limit remains intact.

We note that in general there is an additional operator which mixes with the quark and gluon orbital angular momentum operators,

$$\Delta L = \int d^3x \psi^\dagger (\vec{x}\times(-g\vec{A}))^z \psi. \tag{16}$$

Therefore, we proceed to calculate the matrix element of $\int d^3x \psi^\dagger (\vec{x}\times(-i)\vec{\nabla})^z\psi$ in an ‘‘on-shell’’ quark-gluon-quark state. At the leading-logarithmic order, it contains the following scale-dependent term

$$\frac{\alpha_s}{2\pi}\ln Q^2 \overline{u}(x_2 p)\Big(\not{n} g^{\sigma\perp} + \frac{1}{x_1 - x_2}\ln(x_2/x_1) \times (x_1 \not{n}\gamma^\perp\gamma^\sigma + x_2\not{n}\gamma^\sigma\gamma^\perp)\Big) u(x_1 p)\epsilon_\sigma^* . \tag{17}$$

This result indicates that the operator that mixes with L_q' and L_g' is in fact more complicated than the simple guess ΔL. The most general form is the following non-local operator

$$\int d^3x \vec{x}\times f(n\cdot\partial_\psi, n\cdot\partial_{\psi^\dagger})\psi^\dagger \vec{\gamma}\not{A}\not{n}\psi + \text{H.c.} \tag{18}$$

where ∂_ψ and $\partial_{\psi^\dagger}$ are derivatives acting on ψ and $\psi^\dagger$ respectively, and $f(x,y)$ is a function x and y and takes different forms at different orders of perturbation theory. Therefore, we conclude that to evolve the matrix elements of the gauge-variant orbital angular momentum operators is extremely complicated in the light-cone gauge.[2] The same statement applies to the orbital angular momentum distributions defined in Refs. [14,15].

The evolution in the Feynman gauge is again different. Here we do not have the problem of mixing with infinitely many operators. Apart from the quark and gluon orbital angular momentum operators and ΔL, the ghost field also carries the orbital angular momentum L_ω. Thus, a complete evolution equation will contain at least the mixing of L_q', L_g', L_ω, ΔL among themselves and with $\Delta g, \Delta\Sigma, a_1$ and a_2. Because of its limited use, we have not calculated the

[2]Note that the light-cone gauge calculations must be supplemented with some prescriptions for the light-cone singularities (additional gauge fixing). In our calculation, we have used a prescription such that the regularization is independent of the minus component of the momenta flowing through the gluon propagators. In other regularizations, such as the Mandelstam-Leibbrandt prescription, the result can be different [13]. Of course, for studying truly gauge-invariant quantities, all prescriptions are equivalent.

full mixing matrix. However, we did perform a few calculations just to explore some of differences. We find that the first entry in the evolution matrix in Eq. (15) changes from $-(4/3)C_F$ in the light-cone gauge to $-C_F/3$ in the Feynman gauge. The evolution of L_q' does depend on ΔL

$$\frac{dL_q'}{dt}=\frac{\alpha_s}{2\pi}\left[\left(-\frac{1}{3}C_F+\frac{1}{8}C_A\right)\Delta L-\frac{1}{3}C_F L_q'+\cdots\right]. \quad (19)$$

Conversely, the evolution of ΔL also depends on the other matrix elements

$$\frac{d\Delta L}{dt}=\frac{\alpha_s}{2\pi}\left[-\left(C_F+\frac{1}{8}C_A\right)\Delta L-C_F L_q'+\cdots\right]. \quad (20)$$

These equations would be interesting only if we could find ways to calculate these nonperturbative matrix elements in the Feynman gauge.

If the evolution of the gauge-dependent orbital angular momentum is complicated, how about their experimental measurement? Is it possible, for instance, to have a gauge-invariant extension of the quark orbital angular momentum measurable in high-energy scattering similar to the gluon spin? A gauge-invariant operator that reduces to the quark orbital angular momentum in the light-cone gauge has been discussed recently in Ref. [16]. We note, however, that nonlocal operators with dependence on spatial coordinates have not been seen in factorization of hard forward scattering processes. In particular, inclusive deep-inelastic scattering does not depend on these types of operators.

Given the difficulty of evolving and measuring gauge-dependent orbital angular momenta, a question arises naturally as how to incorporate the polarized gluon distribution $\Delta g(x)$ in unravelling the spin structure of the nucleon, particularly since several experiments have been proposed to measure $\Delta g(x)$ in high-energy processes. A satisfactory solution can be found by following the approach outlined in Ref. [17] and taking seriously the suggestion in Ref. [2].

From the off-forward gluon distributions defined from the twist-two gluon operators, one can introduce the gluon angular momentum distribution [17]

$$J_g(x)=\frac{1}{2}x(g(x)+E_g(x)), \quad (21)$$

where $g(x)$ is the unpolarized gluon distribution and $E_g(x)$ is the forward limit of an off-forward gluon distribution [18]. $J_g(x)$ is gauge invariant, evolves like the twist-two gluon distribution, and is accessible experimentally. From this and the gluon helicity distribution $\Delta g(x)$, we can define the gluon orbital angular momentum distribution

$$L_g(x)=J_g(x)-\Delta g(x). \quad (22)$$

$L_g(x)$ is experimentally measurable because $J_g(x)$ and $\Delta g(x)$ are. The evolution equation for $L_g(x)$ is straightforward

$$\begin{aligned}\frac{d}{d\ln Q^2}L_{gn}&=\gamma_{gg}(n+1)L_{gn}+\gamma_{gq}(n+1)L_{qn}\\&\quad+(\gamma_{gg}(n+1)-\Delta\gamma_{gg}(n))\Delta g_n\\&\quad+\left(\frac{1}{2}\gamma_{gq}(n+1)-\Delta\gamma_{gg}(n)\right)\Delta\Sigma_n, \quad (23)\end{aligned}$$

where γ_{ij} and $\Delta\gamma_{ij}$ are the anomalous dimensions for the spin-independent and spin-dependent twist-two operators [9]. However, the catch here is that $L_g(x)$ can be interpreted as the gluon orbital angular momentum distribution only in the light-cone gauge. If one studies the gluon orbital angular momentum, say in a covariant gauge, $L_g(x)$ would not be sufficient.

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