

## Isospin violation of quark distributions in the Delta (1232)

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1992 J. Phys. G: Nucl. Part. Phys. 18 L167

(<http://iopscience.iop.org/0954-3899/18/9/002>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 111.68.96.57

This content was downloaded on 29/11/2013 at 07:23

Please note that [terms and conditions apply](#).

## LETTER TO THE EDITOR

# Isospin violation of quark distributions in the $\Delta(1232)$

M Ejaz Ayyaz and Pervez Hoodbhoy

Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan

Received 9 March 1992

**Abstract.** Electromagnetism and finite quark mass differences lead to isospin violation of quark distributions in the  $\Delta(1232)$  resonance. We estimate this in a simple valence quark model, and suggest measurement through semi-inclusive deep inelastic lepton scattering from selected nuclear targets.

Isospin-violating nucleon quark distributions are of considerable interest, since they provide a potentially significant test of our understanding of QCD and quark models. Recently there has been some discussion [1, 2] on this subject. Much of this can be applied to other hadrons as well, in particular, the  $\Delta(1232)$  nucleon resonance, which is of considerable interest in nuclear physics. Being a relatively low-lying state, the  $\Delta$  contributes in an important way to various nuclear phenomena. Its transient existence in the deuteron was predicted and confirmed many years ago [3–6]. As an isospin-zero object, the deuteron has no  $\Delta$ -N component; the  $\Delta$ 's must come in pairs. The total  $\Delta$ - $\Delta$  component of the deuteron has been estimated to range from 1% to 3%. Spectator experiments have sought to measure this component by knocking out one  $\Delta$  and detecting the other by its subsequent decay into a nucleon and pion. Various experiments such as these have been discussed by Kisslinger [7].

In this note we enquire into the possibility of another type of spectator experiment:  $e + A \rightarrow e + A' + X$  in the deep inelastic region, i.e.  $Q^2 \rightarrow \infty$ ,  $\nu \rightarrow \infty$ ,  $x = Q^2/2M\nu$  finite. Nucleus  $A$  has isospin zero;  $A'$  has one fewer nucleons and has isospin  $\frac{3}{2}$ . Isospin conservation ensures that the virtual photon has struck a delta in the parent nucleus (figure 1). The purpose of such an experiment would be to measure the scaling structure functions of the  $\Delta$ , and the isospin dependence in the different charge states in particular. One may effectively isolate  $\Delta^{++}$ ,  $\Delta^+$ ,  $\Delta^0$ ,  $\Delta^-$  targets, say in the deuteron, by identifying  $\Delta^-$ ,  $\Delta^0$ ,  $\Delta^+$ ,  $\Delta^{++}$  (in that order) spectators. The advantage of using a nucleus other than the deuteron is obvious; one has a considerably bigger delta component. Candidates for stable targets, used possibly in the form of gas jets, could include  $^{10}\text{B}$ ,  $^{12}\text{C}$ ,  $^{14}\text{N}$ , and  $^{16}\text{O}$ . The  $I = \frac{3}{2}$  spectators are, of course, highly unstable and would have to be identified by their decay products.

We now discuss the light-cone quark distribution in the  $\Delta$ . Imagine for a moment that quark mass differences and charges have been set to zero. The six different valence u and d distributions in various  $\Delta$  isospin states are then expressible in terms

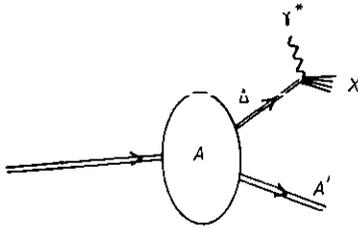


Figure 1.  $\gamma^*$ - $A$  deep inelastic scattering with a spectator nucleus  $A'$  with  $I = \frac{3}{2}$ .

of a single distribution  $a^\Delta(x)$ ,

$$\begin{aligned} u^{\Delta^{++}}(x) &= d^{\Delta^-}(x) = 3a^\Delta(x) & u^{\Delta^+}(x) &= d^{\Delta^0}(x) = 2a^\Delta(x) \\ u^{\Delta^0}(x) &= d^{\Delta^+}(x) = a^\Delta(x). \end{aligned} \quad (1)$$

$a^\Delta(x)$  is normalized according to  $\int dx a^\Delta(x) = 1$ . A single distribution suffices, because all pairs of quarks in the  $\Delta$  ( $S = \frac{3}{2}$ ,  $I = \frac{3}{2}$ ) are coupled to  $S = 1$ . In contrast, the nucleon contains pairs coupled in both  $S = 1$  and  $S = 0$  states, and hence it has two non-identical distributions, say  $u^p(x)$  and  $d^p(x)$ , even when isospin breaking effects are turned off. Thus, at least from a theoretical point of view, the  $\Delta$  is a simpler system for analysing isospin violation.

The spin-averaged quark distribution in a hadron is given by the well-known expression,

$$q(x) = \frac{1}{2P^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \bar{\psi}(0) \gamma^+ \psi(\lambda n) | P \rangle \quad (2)$$

where  $n^\mu$  is a light-cone vector with  $n^2 = 0$ ,  $n^+ = 0$  and  $nP = 1$ . Under the assumption that the state  $|P\rangle$  is composed of three independent constituent quarks,  $q(x)$  can be expressed in terms of the light-cone quark wavefunction  $\phi_+ = \frac{1}{2}(1 + \alpha_3)\phi$  according to

$$q(x) = 2 \int d^3k \delta(x - k^+/P^+) |\phi_+(\mathbf{k})|^2. \quad (3)$$

Here  $k^+ = (1/\sqrt{2})(k^0 + k^3)$  is the light-cone momentum. Clearly a further assumption is needed to relate  $k^0$  to  $\mathbf{k}$ . In a recent work, Sather [2] has used a simple diquark model to estimate quark isospin violations in the nucleon, wherein  $k^0 = M - E_D$ , where  $E_D$  is the energy of the recoiling diquark.  $E_D$  depends on the quark masses as well as on the quark-quark interaction, which in turn contains an electromagnetic component. Assuming that  $\phi_+(\mathbf{k})$  is held constant for differently charged hadrons of the same species, it immediately follows from (3) that

$$\delta q(x) = \frac{1}{M} \left[ \delta M \frac{d}{dx} x q(x) - \delta k^0 \frac{d}{dx} q(x) \right]. \quad (4)$$

Sather [2] has found in a numerical calculation for the nucleon that the above purely kinematic correction dominates over the effect induced by a shift in the wavefunction. We shall assume that this holds equally well for the  $\Delta$ .

Arriving at the  $u$  and  $d$  quark distributions in the  $\Delta$  involves a simple extension of (4), which we perform in two stages. Firstly, imagine that the hyperfine interaction, as well as the quark mass differences and charges, have been 'turned off'. The  $N$  and  $\Delta$  are then degenerate and described by the same  $q(x)$  for both  $u$

and d quarks. Upon turning on the hyperfine interaction, the distribution of quarks in the  $\Delta$  (see (1)) becomes

$$a^\Delta(x) = q(x) + \frac{1}{M} \left[ (M_\Delta - M) \frac{d}{dx} xq(x) - (E_\Delta - E) \frac{d}{dx} q(x) \right] \quad (5)$$

approximately. Here  $M \approx 1085$  MeV is the mean N- $\Delta$  mass, and  $E_\Delta - E \approx \frac{1}{3}(M_\Delta - M)$  is the shift in the single quark energy on account of gluon exchange with the spectator diquark. Restoring charges and quark mass differences leads to a splitting of  $a^\Delta(x)$  into six unequal distributions  $q_i^{\Delta j}(x)$ , where  $j = ++, +, 0, -$  are the  $\Delta$  charge states and  $i = u, d$ . Again using (4),

$$q_i^{\Delta j}(x) = N_i^j a^\Delta(x) + N_i^j \frac{1}{M_\Delta} \left[ \delta M^{\Delta j} \frac{d}{dx} x a^\Delta(x) - \delta E_i^{\Delta j} \frac{d}{dx} a^\Delta(x) \right]. \quad (6)$$

Here  $\delta M_{\Delta j} = M_{\Delta j} - M_\Delta$  is the mass difference between different  $\Delta$ 's,  $\delta E_i^{\Delta j} = E_i^{\Delta j} - E^\Delta$  is the shift in the single quark energy, and  $N_i^j$  are normalization constants:  $N_u^{++} = N_d^- = 3$ ,  $N_u^+ = N_d^0 = 2$ ,  $N_u^0 = N_d^+ = 1$  and  $N_u^- = N_d^{++} = 0$ . In this simple model,  $\delta E_i^{\Delta j}$  is easily estimated:

$$\delta E_i^{\Delta j} = \delta m_i + Z_i Z_i^j \langle e^2/R \rangle. \quad (7)$$

Here  $Z_i$  is the charge of quark  $i$ ,  $Z_i^j$  the charge of the recoiling diquark, and  $\delta m_i = m_i - m$ , where  $m = \frac{1}{2}(m_u + m_d)$ . Similarly, the mass differences within the multiplet are

$$\delta M^{\Delta j} = \sum N_i^j \delta m_i + C^j \langle e^2/R \rangle \quad (8)$$

where  $C^{++} = \frac{4}{3}$ ,  $C^+ = 0$ ,  $C^0 = -\frac{1}{3}$ , and  $C^- = \frac{1}{3}$  are coefficients of the Coulomb energy term. From the known quark mass differences,  $\delta m_d = -\delta m_u = \frac{1}{2}(m_d - m_u) \approx 2.15$  MeV. The relation analogous to (8) for the nucleon is

$$M_n - M_p = (m_d - m_u) - \frac{1}{3} \langle e^2/R \rangle. \quad (9)$$

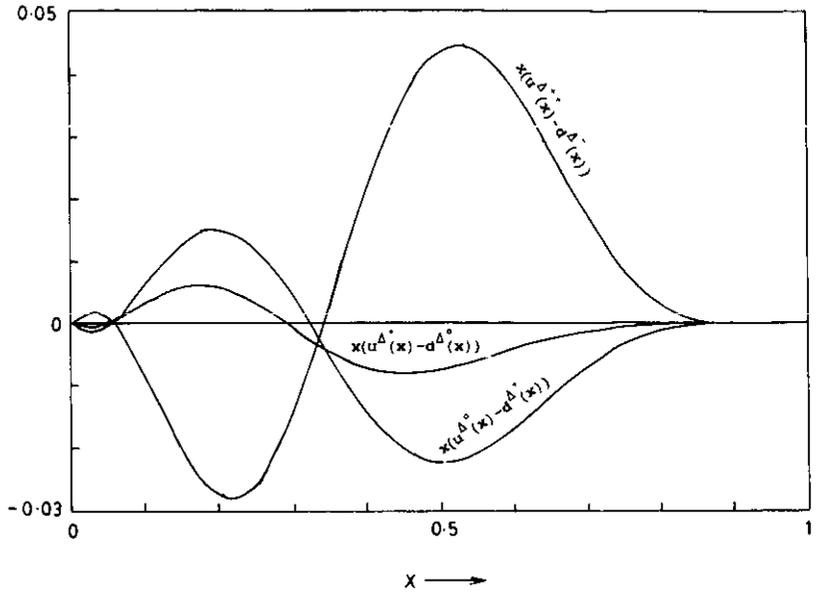
Using the nucleon mass difference gives  $\langle e^2/R \rangle$ . Thus, there are no free parameters and the  $q_i^{\Delta j}(x)$  may be directly worked out. With  $q(x)$  taken from a bag model calculation [8], the results for the isospin-violating structure functions of the  $\Delta$  at the bag scale ( $Q_0^2 \approx 0.45^2$  GeV<sup>2</sup>) are shown in figure 2. Evolution to higher values of  $Q^2$  is performed as in [2]—aside from the normal evolution coming from gluon radiation, one must include first-order corrections arising from photon radiation,

$$[\delta a_i^{\Delta j}(t)]_n + e^{\langle d \rangle A_n} [\delta a_i^{\Delta j}(0)]_n + \frac{3}{8\pi} e_i^2 \alpha_{EM} A_n t a^\Delta(t). \quad (10)$$

In the above,  $A_n$  is the anomalous dimension for the  $n$ th moment and  $\langle d \rangle \approx 0.5$ . The larger coupling of the photon with the u quarks relative to d quarks would cause isospin breaking from radiative corrections even if isospin symmetry held at some initial scale. The evolved distributions are shown in figure 3.

Having estimated the isospin violation in the isolated deltas, it is necessary to see how this is reflected in the structure function of the nucleus which contains them as virtual particles. By way of example we consider the deuteron. The quark distributions can be obtained by the usual convolution procedure,

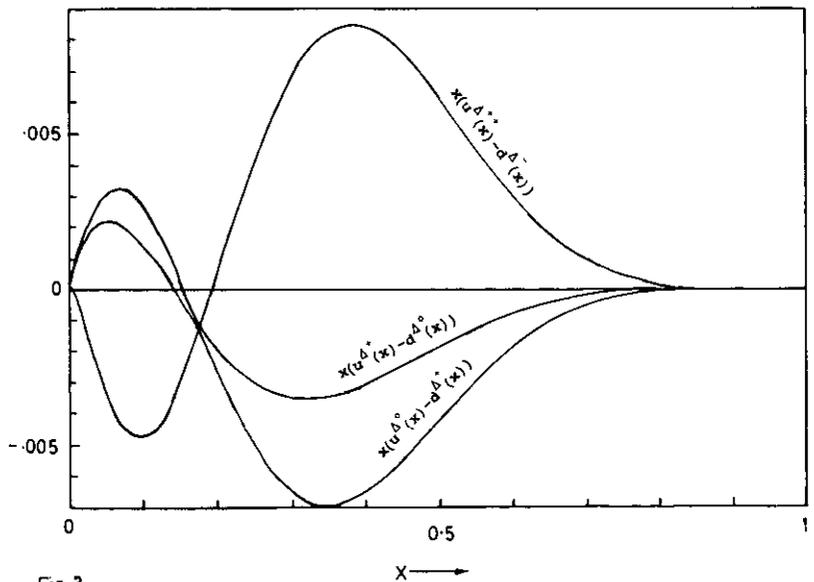
$$q_i^{D, \Delta j} = q_i^{\Delta j} \otimes f^{D, \Delta j} \quad (11)$$



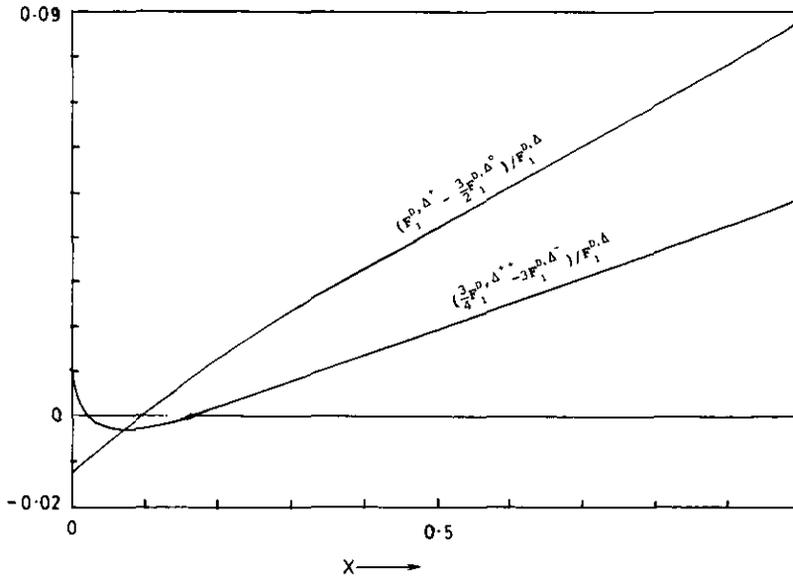
**Figure 2.** Isospin asymmetries in various delta charge states at the constituent scale ( $t=0$ ).

where  $f^{D,\Delta_j}(y)$  is the light-cone distribution for finding the  $\Delta_j$  with momentum fraction  $y$  of the deuteron. In the absence of isospin violation, one would have  $f^{D,\Delta_j} = f^{D,\Delta}$  for all charge species, where

$$f^{D,\Delta}(y) = \frac{1}{4} \int d^3 p_\Delta \delta(y - (p_\Delta^+ / P^+)) (1 + (p_\Delta^3 / M_\Delta)) f^{D,\Delta}(p_\Delta). \quad (12)$$



**Figure 3.** Isospin asymmetries, as in figure 2, but evolved to  $Q^2 = 12.6 \text{ GeV}^2$  with both QCD and QED interactions.



**Figure 4.** Charge-weighted difference ratios in the  $\Delta$  component of the deuteron structure functions at  $Q^2 = 12.6 \text{ GeV}^2$ .

For the wavefunction in [5], the momentum probability  $f^{D,\Delta}(\mathbf{p}_\Delta)$  is normalized according to  $\int d^3p_\Delta f^{D,\Delta}(\mathbf{p}_\Delta) \approx 0.01$ . However, once different  $\Delta$  masses and charges are admitted, the fraction of momentum carried by each species will change. Isospin violation in  $f$  will also occur,

$$f^{D,\Delta j} = f^{D,\Delta} + \delta f^{D,\Delta j}. \quad (13)$$

By taking the appropriately charge-weighted difference between  $F_1^{D,\Delta^{++}}$  and  $F_1^{D,\Delta^-}$ , as well as between  $F_1^{D,\Delta^+}$  and  $F_1^{D,\Delta^0}$ , we can eliminate isospin violation in the  $\Delta$  momentum distribution due to charge effects. The effect due to mass is calculated from (12) from the shift in the argument of the  $\delta$  function, exactly as before.

Numerical results in figure 4 show two appropriately charge-weighted isospin-violating structure function differences, divided by the averaged sum of delta contributions to the deuteron. As expected from the nucleon case, the violation is at the level of a few percent. Its measurement would be interesting from a theoretical point of view, given the relatively simpler nature of the isospin-violating mechanism in the delta. Although our model is admittedly crude, it has no free parameters and may be sufficiently accurate as a first estimate.

This research was supported by the PAEC program for research in Pakistani universities. PH thanks Leonard Kisslinger for a valuable discussion.

## References

- [1] Signal A I and Thomas A W 1989 *Phys. Rev. D* **38** 2832
- [2] Sather E 1991 *MIT preprint CTP 2019* (submitted to *Phys. Lett. B*)
- [3] Kerman A K and Kisslinger L S 1969 *Phys. Rev.* **180** 1483
- [4] Kisslinger L S 1974 *Phys. Lett.* **48B** 410

- [5] Nath N R and Weber H J 1972 *Phys. Rev. B* **6** 1975
- [6] Benz P and Soding P 1974 *Phys. Lett.* **52** 367
- [7] Kisslinger L S 1979 Experimental Tests of isobar components of Nuclei *Mesons in Nuclei* vol 1 ed Mannque Rho and Denys Wilkinson (Amsterdam: North-Holland) p 261
- [8] Bensch C J and Miller G A 1988 *Phys. Lett.* **215B** 381