Nucleon-Quarkonium Elastic Scattering and the Gluon Contribution to Nucleon Spin

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It is shown that the amplitude for the scattering of a heavy quarkonium system from a nucleon near threshold is completely determined by the fraction of angular momentum, as well as linear momentum, carried by gluons in the nucleon. A form for the quarkonium-nucleon nonrelativistic potential is derived. [S0031-9007(99)09380-1]

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Heavy quark-antiquark systems are essentially hydrogenic, have a size that varies in inverse proportion to the heavy quark mass, and therefore have an interaction with light hadrons that can be systematically and rigorously analyzed using QCD and the operator product expansion [1–4]. The possibility that the attractive force between the $c\bar{c}$ system and nucleons may be strong enough to create a bound state has been speculated upon for some time now [5–7]. More recently, Brodsky and Miller [8] have confirmed that those various contaminations to the nucleon-J/ψ force that are not computable from first principles are numerically small. They have also suggested using the exclusive reaction $\pi^+ + D \to J/\psi + p + p$ as a means to investigate low-energy nucleon-J/ψ elastic scattering.

In this Letter we suggest that slightly off-forward nucleon-J/ψ elastic scattering can be used to probe the gluon angular momentum content of the nucleon. This follows the proposal by Ji [9] who suggested using deeply virtual Compton scattering as a means of measuring the quark orbital contribution to the nucleon's spin.

We begin with the covariant version of the expression derived by Bhanot and Peskin [3] for the scattering amplitude

$$\mathcal{M} = \frac{1}{2}\sum_{n=2}^{\infty} d_n a_0^3 \varepsilon_0^{2-n} v_{\mu_1}\cdots v_{\mu_n}\langle p'|O^{\mu_1\cdots\mu_n}|p\rangle. \quad (1)$$

In the above, n runs over the even integers, ε_0 is the binding energy of the quarkonium, a_0 is its the Bohr radius, and d_n are exactly calculable coefficients [3] in the large m_Q limit. The operator $O^{\mu_1\cdots\mu_n}$ is

$$O^{\mu_1\cdots\mu_n} = F^{\mu_1\rho} iD^{\mu_2}\cdots iD^{\mu_n} F_\rho^{\ \mu_n}. \quad (2)$$

From this we are led to define

$$T^{\mu_1\cdots\mu_n} = F^{(\mu_1\rho}\overleftrightarrow{iD^{\mu_2}}\cdots\overleftrightarrow{iD^{\mu_n}}F_\rho^{\ \mu_n)}. \quad (3)$$

The brackets refer to symmetrization in all n indices and subtraction of traces. This procedure makes the above operator, which could be called the generalized gluon stress-energy tensor, transform according to a definite representation of the Lorentz group and have twist equal to two. The matrix elements of $T^{\mu_1\cdots\mu_n}$ and $O^{\mu_1\cdots\mu_n}$ between hadron states are equal up to terms of $O(m/E)$ where E is the energy of the proton in the quarkonium rest frame. For infinitely massive quarks, the 4-velocity of the quarkonium v^μ is conserved in the scattering although its momentum is not.

The matrix element of the traceless tensor $T^{\mu_1\cdots\mu_n}$ taken between states of unequal momentum can be written in complete generality as [10]

$$\begin{aligned}\langle p'|T^{\mu_1\cdots\mu_n}|p\rangle = {} & \bar{u}(p's')\gamma^{(\mu_1}u(ps)\sum_{i=0}^{[(n-1)/2]} A_{n,2i}(t)\Delta^{\mu_2}\cdots\Delta^{\mu_{2i+1}}P^{\mu_{2i+2}}\cdots P^{\mu_n)} \\ & + \bar{u}(p's')\frac{\sigma^{(\mu_1\alpha}i\Delta_\alpha}{2m}u(ps)\sum_{i=0}^{[(n-1)/2]} B_{n,2i}(t)\Delta^{\mu_2}\cdots\Delta^{\mu_{2i+1}}P^{\mu_{2i+2}}\cdots P^{\mu_n)} \\ & + \bar{u}(p's')u(ps)\frac{1}{m}C_n(t)\Delta^{(\mu_1}\cdots\Delta^{\mu_n)}. \end{aligned} \quad (4)$$

The last term exists for even n only. In the above,

$$P^\mu = \frac{1}{2}(p' + p)^\mu, \quad (5)$$

$$\Delta^\mu = (p' - p)^\mu, \quad (6)$$

$$t = \Delta^2. \quad (7)$$

While our discussion has been couched in a covariant language, the greatest insight is obtained by working in the quarkonium rest frame. Considering the quarkonium to be infinitely massive, the nucleon scatters elastically ($E' = E = \sqrt{\vec{p}^{\,2} + m^2}$) through an angle θ in terms of which

$$t = -4\vec{p}^{\,2}\sin^2(\theta/2). \quad (8)$$

We shall be interested in only the special kinematic region where θ is sufficiently small so that $t \ll m^2$. Under

the assumption that the matrix elements of $T^{\mu_1\cdots\mu_n}$ and $O^{\mu_1\cdots\mu_n}$ are approximately equal, which will be true if $\gamma = E/m \gg 1$, a simple calculation yields

$$\mathcal{M} = \frac{1}{2} a_0^3 \varepsilon_0^2 \sum_{n=2}^{\infty} d_n \left(\frac{\gamma m}{\varepsilon_0}\right)^n \{2A_{n0} + \gamma B_{n0} \langle i\sigma \cdot \hat{n}\rangle \theta\}, \tag{9}$$

where

$$\hat{n} = \frac{\vec{p}' \wedge \vec{p}}{\| \vec{p}' \wedge \vec{p} \|} \tag{10}$$

is the vector normal to the scattering plane and $A_{n0} \equiv A_{n0}(t = 0)$, $B_{n0} \equiv B_{n0}(t = 0)$. The quantity $\langle i\sigma \cdot \hat{n}\rangle$ is ± 1 if the initial and final nucleon spins are flipped and zero otherwise, while the first term (proportional to A_{n0}) vanishes for a spin flip.

In principle, Eq. (9) allows for the extraction of A_{n0} and B_{n0}, which are quantities of significant physical importance. From Eq. (4) it follows that A_{n0} is the nth moment of the usual (forward) gluon distribution $G(x)$,

$$A_{n0} = \int_{-1}^{+1} dx\, x^{n-1} G(x). \tag{11}$$

It is evident that A_{20} has special significance as the momentum fraction carried by gluons in a proton ($A_{20} \approx 0.5$). To see the significance of B_{n0}, define a gluon angular momentum from the following generalization of the gluon angular momentum tensor,

$$\mathbf{M}^{\alpha\beta\mu_1\cdots\mu_n}(x) = x^\alpha O^{\beta\mu_1\cdots\mu_n} - x^\beta O^{\alpha\mu_1\cdots\mu_n} - \text{traces}. \tag{12}$$

There is only one "reduced matrix element" of the above, denoted by J_n below [10]

$$\langle PS| \int d^4x\, \mathbf{M}^{\alpha\beta\mu_1\cdots\mu_n}(x) |PS\rangle = 2J_n \frac{2S_\rho P_\sigma}{(n+1)m^2} \{\varepsilon^{\alpha\beta\rho\sigma} P^{\mu_1} \cdots P^{\mu_n} + \cdots\}, \tag{13}$$

where the ellipses denote antisymmetrization with respect to the indices α, β and subtraction of traces. One can therefore *define* a gluon angular momentum distribution $J(x)$ through

$$J_n = \int_{-1}^{+1} dx\, x^{n-1} J(x). \tag{14}$$

Inserting Eq. (12) into Eq. (13) after taking the forward limit appropriately yields

$$J_n = \frac{1}{2}(A_{n0} + B_{n0}). \tag{15}$$

In particular, for $n = 2$, the sum $A_{20} + B_{20}$ is twice the angular momentum carried by gluons in the nucleon.

Unfortunately, the kinematic conditions required for extracting all the B_{n0} from Eq. (9) are unachievable—it is difficult to see how one can keep $t \ll m^2$ while maintaining $\gamma \gg 1$, which is necessary if the matrix elements of difference $T^{\mu_1\cdots\mu_n} - O^{\mu_1\cdots\mu_n}$ are to be ignorable. However, for $n = 2$, it is possible to use the trick of the QCD scale anomaly [6,7] to calculate the trace part. Essentially this consists of writing the matrix element of the dominant interaction operator at the threshold energy using

$$\langle p'|E \cdot E|p\rangle = v_{\mu_1} v_{\mu_2} \langle p'|T^{\mu_1\mu_2}|p\rangle - \frac{g^3}{2\beta(g)} \langle p'|\theta^\mu_\mu|p\rangle. \tag{16}$$

All gluon colors are summed over in the product of the two color electric fields, and the scale anomaly has been used to relate the trace of the stress-energy tensor $\theta^{\mu\nu}$ with the square of the gluon field strength and the QCD beta function $\beta(g)$,

$$\partial_\mu j_D^\mu = \theta^\mu_\mu = \frac{\beta(g)}{2g^3} F^{\mu\nu} F_{\mu\nu}, \tag{17}$$

$$\beta(g) = -\frac{9g^3}{16\pi^2}. \tag{18}$$

A straightforward calculation [keeping only the $n = 2$ term in Eq. (1)] yields the following simple expression for the small angle scattering amplitude:

$$\mathcal{M} = \frac{1}{6} d_2 m^2 a_0^3 \left[\frac{16\pi^2}{9} + 2A_{20}\left(\gamma^2 - \frac{1}{4}\right)\right] + \frac{1}{6} d_2 m^2 a_0^3 (\gamma - 1)\left[\left(\gamma + \frac{1}{4}\right) A_{20} + \gamma(\gamma + 1) B_{20}\right] \langle i\sigma \cdot \hat{n}\rangle \theta. \tag{19}$$

In the above $d_2 = 28\pi/27$. This formula is valid provided the velocity of the nucleon in the quarkonium rest frame is small [so that all but the first term in Eq. (1) can be discarded], and the scattering angle θ is also sufficiently small. The normalization of the first term agrees with that of Kaidalov and Volkovitsky [6] but disagrees with that of Luke, Manohar, and Savage [7]. Brodsky and Miller [8] essentially propose to test the forward ($\theta = 0$) part of the above equation using the exclusive reaction $\pi^+ + D \to J/\psi + p + p$.

One can ask what nonrelativistic nucleon-quarkonium potential $V_{\Phi p}(\vec{r})$ will reproduce the scattering amplitude $\mathcal{M}$ close to threshold ($\gamma \to 1$) when inserted into the Schrodinger equation and used in the Born approximation.

It is easily seen that $V_{\Phi p}(\vec{r})$ below fulfills this,

$$V_{\Phi p}(\vec{r}) = V_1(r) + \vec{\nabla} V_2(r) \cdot (i\vec{\sigma} \wedge \vec{\nabla}), \tag{20}$$

where $V_1(r)$ and $V_2(r)$ are constrained to obey

$$\int_0^\infty dr\, r^2 V_1(r) = \frac{d_2 m a_0^3}{48\pi}\left(\frac{16\pi^2}{9} + \frac{3}{2} A_{20}\right), \tag{21}$$

$$\int_0^\infty dr\, r^2 V_2(r) = \frac{d_2 a_0^3}{48\pi m}\left(\frac{5}{8} A_{20} + B_{20}\right). \tag{22}$$

We stress that no model parameters are involved; the above is a rigorous result of QCD in the $m_Q \to \infty$ limit.

To conclude, it was shown that the amplitude for the scattering of a heavy quarkonium system from a nucleon at threshold is completely determined by the fraction of angular momentum, as well as linear momentum, carried by gluons in the nucleon. Totally exclusive experiments involving the elastic low-energy scattering of quarkonium from nuclei may therefore be a means of investigating the gluonic angular momentum component of nucleons. Although going from nucleons to nuclei necessarily brings in the nuclear wave function, for light enough nuclei like the deuteron this wave function is probably sufficiently well known to embark on a meaningful experiment.

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