Probing Quark-Distribution Amplitudes through Generalized Parton Distributions at Large Momentum Transfer

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In the large momentum transfer limit, generalized parton distributions can be calculated through a QCD factorization theorem which involves perturbatively calculable hard kernels and light-cone parton distribution amplitudes of hadrons. We illustrate this through the $H_q(x, \xi, t)$ distribution for the pion and the proton, presenting the hard kernels at leading order. As a result, experimental data on the generalized parton distributions in this regime can be used to determine the functional form of the parton distribution amplitudes which has thus far been quite challenging to obtain. Our result can also be used as a constraint in phenomenological generalized parton distribution parametrizations.

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It has been established in perturbative quantum chromodynamics (pQCD) that hard exclusive processes in the asymptotic limit depend on the nonperturbative light-cone parton distribution amplitudes of hadrons [1–4]. However, the functional form of these amplitudes in parton momenta x_i has been very difficult to determine either from experimental data or from theoretical calculations [5,6]. For instance, the asymptotic electromagnetic form factors depend only on an integral of the distribution amplitudes [1,3]. In the pion-nucleus diffractive production of two jets, one can in principle learn about the shape of the quark distribution amplitude in the pion [7]. Poor knowledge of the distribution amplitude in the proton has been the main obstacle in deciding at what momentum transfer the asymptotic pQCD calculation is relevant [8–11].

In this Letter, we study the generalized parton distributions (GPDs) [12,13] of hadrons in the large momentum transfer limit. The GPDs are a new class of hadron observables which combine the physics of electromagnetic form factors and Feynman parton distributions, and are related to quantum phase-space distributions of the partons through Fourier transformation [14]. Apart from the renormalization scale, they depend on the momentum transfer t as in a form factor, light-cone momentum x as in a parton distribution, and the projection of the momentum transfer along the light-cone direction ξ, also known as the skewness parameter. The GPDs can be computed in the light-cone wave function overlap framework in principle for any value of t [15].

We report here that the GPDs in the $-t \to \infty$ limit are calculable through QCD factorization in which the nonperturbative physics is included in the light-cone distribution amplitudes of hadrons. Using this, the functional form of the distribution amplitudes can be studied through the GPDs' dependence on x and ξ. Conversely, our result provides a constraint on phenomenological GPD parametrizations. The GPDs at large t can be measured, for example, from deeply virtual Compton scattering, hard exclusive meson production, or doubly virtual Compton scattering in the kinematic regime $Q^2 \gg -t \gg \Lambda_{\rm QCD}^2$ in which the factorization theorems for scattering amplitudes have been proven [16]. However, it can be experimentally challenging to measure the cross sections in this regime because of additional power suppression in t; we will not explore this issue here.

We illustrate our main point first by considering the generalized parton distribution $H(x, \xi, t)$ for the pion, defined through

$$H_q(x, \xi, t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\left\langle \pi; P' \left| \overline{\psi}_q\left(-\frac{\lambda}{2}n\right) \not{n} \mathcal{P} e^{-ig\int_{\lambda/2}^{-\lambda/2} d\alpha n\cdot A(\alpha n)} \psi_q\left(\frac{\lambda}{2}n\right)\right| \pi; P\right\rangle, \quad (1)$$

where P and P' are the initial and final state pion momenta, respectively, $t = (P - P')^2$, and $\mathcal{P}$ indicates path ordering for the light-cone gauge link. Introducing $\overline{P} = (P + P')/2$ along the z direction and the conjugation light-cone four-vector n, such that $n^2 = 0$ and $n \cdot \overline{P} = 1$, the skewness parameter is the projection of the momentum transfer $P' - P$ along $\overline{P}$ direction, $\xi = -n \cdot (P' - P)/2$. The initial and final light-cone momenta of the quarks are then $n \cdot k = x + \xi$ and $n \cdot k' = x - \xi$, respectively.

In the large momentum transfer limit, one can calculate the above GPD using the pQCD factorization formalism which has been widely applied to electromagnetic and other form factors [1–3,17]. The leading pQCD contribution is shown in Fig. 1, where the initial and final pion states are replaced by the light-cone Fock component with the minimal number of partons. The circled crosses in the diagrams represent the bilocal quark operator in Eq. (1). The hard part responsible for the large momentum

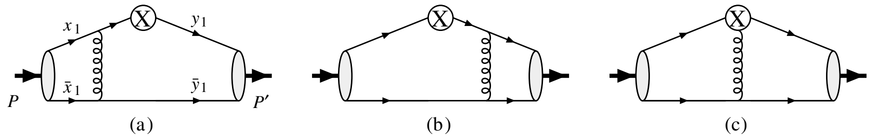

FIG. 1. Leading pQCD diagrams contributing to the pion's generalized parton distribution $H(x, \xi, t)$ at large $-t$. The circled crosses represent the nonlocal quark operator.

transfer contains a single gluon exchange just like in the electromagnetic form factor. In the first two diagrams 1(a) and 1(b), there is a hard gluon exchange between the two quark lines, and in the third one, there is a gluon coming from the gauge link. Since the transverse momenta of the quarks are expected to be on the order of $\Lambda_{\rm QCD}$, we may ignore them in calculating the hard part. Thus we can effectively integrate out $k_\perp$ in the pion wave function to obtain the distribution amplitude, $\phi(x) = \int \frac{d^2k_\perp}{(2\pi)^3}\psi(x, k_\perp)$. The parton transverse momenta flowing into the hard part are now taken to be zero.

The result of the above pQCD analysis is a factorization formula for the GPD at large t in terms of the quark distribution amplitude

$$H_q(x, \xi, t, \mu) = \int dx_1 dy_1 \phi^*(y_1, \mu)\phi(x_1, \mu) \times T_{Hq}(x_1, y_1, x, \xi, t, \mu), \quad (2)$$

where T_{Hq} is the hard part and can be calculated as a perturbation series in α_s. All quantities in the above equation depend on the renormalization scale μ. The μ dependence in the hard part must be such that it accounts for the difference between the GPD and the distribution amplitude.

The leading contribution to the hard part can be calculated straightforwardly,

$$T_u(x, x_1, y_1) = \frac{4\pi\alpha_s C_F}{\bar{x}_1\bar{y}_1(-t)}\delta(x - \lambda_1)\left[(1-\xi) + \frac{1-\xi^2}{\lambda_1 - \tilde{\lambda}_1}\right] + \text{H.c.}, \quad (3)$$

where $C_F = 4/3$, H.c. stands for a term obtained by exchange x_i and y_i, and ξ and $-\xi$, $\lambda_1 = y_1 + \bar{y}_1\xi$, $\tilde{\lambda}_1 = x_1 - \bar{x}_1\xi$ ($\bar{x} = 1 - x$). Since $0 < y_1 < 1$, the first term contributes when $x > \xi$, whereas the second term contributes when $x > -\xi$, which indicates an up-antiup pair contribution. The antiquark is generated through the one-gluon exchange on the top of the valence wave function. This corresponds to the $n \neq n'$ Efremov-Radyushkin-Brodsky-Lepage diagrams' contribution to the GPDs [15]. The GPD for the down quark $H_d(x, \xi, t)$ can be obtained from that of the up quark through simple charge symmetry, $H_d(x, \xi, t) = -H_u(-x, \xi, t)$.

The above result can be translated into one for the moments of the GPDs $H_q^{(n)}(\xi, t) = \int_{-1}^{1} dx x^{n-1} \times H_q(x, \xi, t)$. In fact, the factorization formula applies for the individual moments, $H_q^{(n)}(\xi, t) = \int dx_1 dy_1 \phi^*(y_1) \times \phi(x_1) T_{Hq}^{(n)}(x_1, y_1, \xi, t)$, where $T_{Hq}^{(n)}(x_1, y_1, \xi, t)$ is simply the nth moment of $T_{Hq}(x_1, y_1, x, \xi, t)$. For the up quark in the pion, we have

$$T_u^{(n)}(\xi, t) = \frac{4\pi\alpha_s C_F}{\bar{x}_1\bar{y}_1(-t)}\left[(1-\xi)\lambda_1^{n-1} + (1+\xi)\tilde{\lambda}_1^{n-1} + (1-\xi^2)\sum_{m=0}^{n-2}\lambda_1^m\tilde{\lambda}_1^{n-m-2}\right], \quad (4)$$

which contains both even and odd powers of ξ. For $n = 1$, the above reproduces the hard part in the QCD factorization formula for the pion form factor [1–3]. For $n = 2$, $T_u^{(2)}(\xi, t) = 4\pi\alpha_s C_F/(\bar{x}_1\bar{y}_1 t)[(x_1 + y_1 + 1) + 2(x_1 - y_1)\xi + (x_1 + y_1 - 3)\xi^2]$. It is easy to see that the linear dependence in ξ does not contribute to $H_u^{(1)}(\xi, t)$ because of the symmetry in the initial and final states. For the same reason, all odd powers of ξ in $T_u^{(n)}$ do not contribute to the GPD moments.

It has been suggested from the dijet production [7] and $\gamma^*\gamma \to \pi$ transition [18] that the pion distribution amplitude at $\mu > 2$ GeV is very close to the asymptotic amplitude $\sqrt{6}f_\pi x(1-x)$ [1–3], where $f_\pi = 93$ MeV. If so, we can make the prediction for the H_u as follows:

$$H_u(x, \xi, t) = \frac{16\pi\alpha_s f_\pi^2}{(-t)}\left\{\theta(x-\xi)\frac{(x-\xi)}{(1-\xi)} \times \left[-1 - \frac{(x+\xi)}{(1+\xi)}\log\frac{(1-x)^2}{(x+\xi)^2}\right] + (\xi \to -\xi)\right\}, \quad (5)$$

which is continuous at $x = \xi$ and $x = -\xi$. The quantity in the braces is a profile function and is plotted for four different ξ in Fig. 2. The function diverges at $x = 1$, indicating the breaking down of $1/t$ expansion. This divergence generates a slow decrease of the GPD moments at large n, and is present even when $\xi = 0$. We note that the limit $x \to 1$ and $-t \to \infty$ may not be interchangeable. If we take the limit $x \to 1$ first, H_q may not vanish in the subsequent $-t \to \infty$ limit because the pion momentum is now carried by a single quark.

Now we turn to the proton case. The factorization formula for the GPD H_q takes a similar form

$$H_q(x, \xi, t) = \int [dx][dy]\Phi_3^*(y_1, y_2, y_3)\Phi_3(x_1, x_2, x_3) \times T_{Hq}(x_i, y_i, x, \xi, t), \quad (6)$$

where $[dx] = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$, and $\Phi_3(x_i)$ is the three-quark distribution amplitude [19].

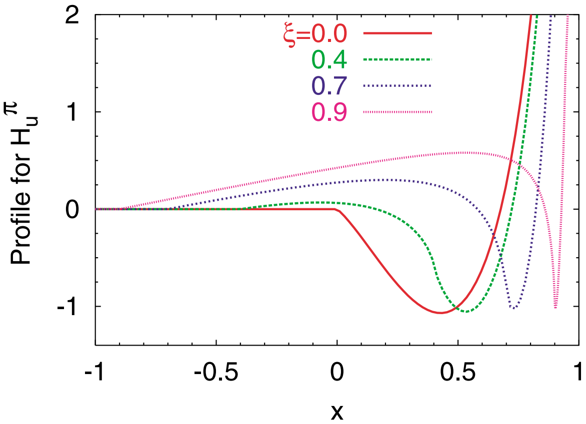

FIG. 2 (color online). The profile function of $H(x, \xi, t)$ for the pion in the aymptotic limit.

In the leading order in α_s, there are three classes of diagrams contributing to the hard part, each with a representative shown in Fig. 2. The first class consists of diagrams with two-gluon exchanges not attached to the nonlocal operators. Shown in Fig. 3(a) is one of the 14 possible diagrams in this class. The second class has one gluon coming from the gauge link in the nonlocal operator, and the third class has two gluons. To calculate the hard part, we arrange the first quark to have spin up, the second spin down, and the third spin up again, with Feynman momentum x_1, x_2, and x_3 for the incoming quarks, and y_1, y_2, and y_3 for outgoing quarks, respectively. We use T_i to denote the hard part with the nonlocal operator inserted on the line i. Then the hard part from the proton is

$$T_u^p = \tfrac{1}{3}(2T_1 + T_2 + T_3 + T_1' + T_3'),$$
$$T_d^p = \tfrac{1}{3}(T_2 + T_3 + T_2'). \tag{7}$$

T_i' is obtained from T_i by interchanging y_1 and y_3.

Our result for the hard part is

$$T_1 = \frac{2\pi^2 C_B^2 \alpha_s^2}{t^2}\left\{K_{11}\delta(x-\lambda_1) + K_{12}\frac{\delta(x-\lambda_1)-\delta(x-\tilde{\lambda}_1)}{\lambda_1 - \tilde{\lambda}_1} + K_{13}\frac{\delta(x-\lambda_1)-\delta(x-\eta_1)}{\lambda_1-\eta_1}\right.$$
$$\left. + K_{14}\frac{\delta(x-\lambda_1)-\delta(x-\eta_1)}{(\lambda_1-\eta_1)(\eta_1-\tilde{\lambda}_1)}\right\} + \text{H.c.}, \tag{8}$$

$$T_2 = \frac{2\pi^2 C_B^2 \alpha_s^2}{t^2}\left\{K_{21}\delta(x-\eta_2) + \left[K_{13}'\frac{\delta(x-\lambda_2)-\delta(x-\eta_2)}{\lambda_2-\eta_2} + (1\leftrightarrow 3)\right] + K_{14}'\frac{\delta(x-\lambda_2)-\delta(x-\eta_2)}{(\lambda_2-\eta_2)(\eta_2-\tilde{\lambda}_2)}\right\} + \text{H.c.}, \tag{9}$$

where $C_B = 2/3$, $\lambda_i = y_i + \bar{y}_i\xi$, $\eta_1 = 1 - x_3 - y_2 + (y_2 - x_3)\xi$, and $\eta_2 = \eta_1(1 \leftrightarrow 2)$. The functions K_{ij} are defined as

$$K_{11} = \frac{1}{x_3 y_3 \bar{x}_1^2 \bar{y}_1^2} + \frac{1}{x_2 y_2 \bar{x}_1^2 \bar{y}_1^2} - \frac{1}{x_2 y_2 x_3 y_3 \bar{x}_3 \bar{y}_1}, \qquad K_{12} = \frac{1-\xi}{x_2 y_2 \bar{x}_1^2 \bar{y}_1} + \frac{1-\xi}{x_3 y_3 \bar{x}_1^2 \bar{y}_1}, \qquad K_{13} = \frac{1-\xi}{x_2 y_2 x_3 y_3 \bar{x}_3},$$
$$K_{14} = \frac{(1+\xi)(1-\xi)}{x_2 y_2 x_3 y_3}, \qquad K_{21} = \frac{1}{x_1 y_1 x_3 y_3 \bar{x}_3 \bar{y}_1}, \tag{10}$$

and $K_{ij}' = K_{ij}(1 \leftrightarrow 2)$. T_3 can be obtained from T_1 by exchanging (x_1, y_1) and (x_3, y_3), respectively. From the above, we can calculate the GPD moments for the nucleon in a factorization form. If we take the first moment, we recover the pQCD prediction for the Dirac form factor $F_1(Q^2)$ [1–3]. If we take the second moment, we find the pQCD prediction for gravitational form factors $A(Q^2)$ and $C(Q^2)$ at large Q^2 [12]. The contribution to $C(Q^2)$ is zero at this order in $1/t$. This is because $C(Q^2)$ also contribute to the helicity-flip GPD $E(x, \xi, t)$ which is subleading in the large $-t$ limit.

One can make a numerical calculation of $H_u(x, \xi, t)$ using various model amplitudes in the literature [3,5,10]. Using the strategy of Ref. [20], we have computed $t^2 H_u$, shown in Fig. 4, for three different ξ at $t^2 = -20\ \text{GeV}^2$ with the asymptotic, Chernyak-Zhitnitsky (CZ), and Gari-Stefanis (GS) amplitudes. Although the CZ and GS amplitudes both give a reasonable account of data on F_1^p for $-t \gtrsim 10\ \text{GeV}^2$, the two yield very different predictions for the GPD. Note that the scale of H_u is strikingly large; a relatively small Dirac F_1 results from the cancellation in the integration.

In summary, we have obtained a QCD factorization formula for the generalized parton distributions in terms of the nonperturbative light-cone distribution amplitudes

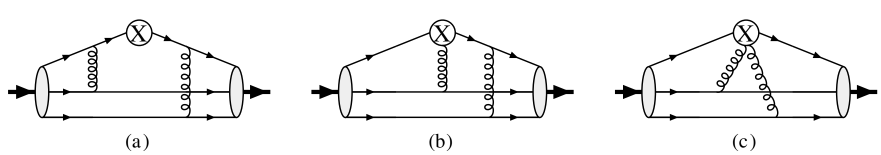

FIG. 3. Representatives from three classes of QCD diagrams contributing to the proton GPD $H_q(x, \xi, t)$ in the asymptotic limit.

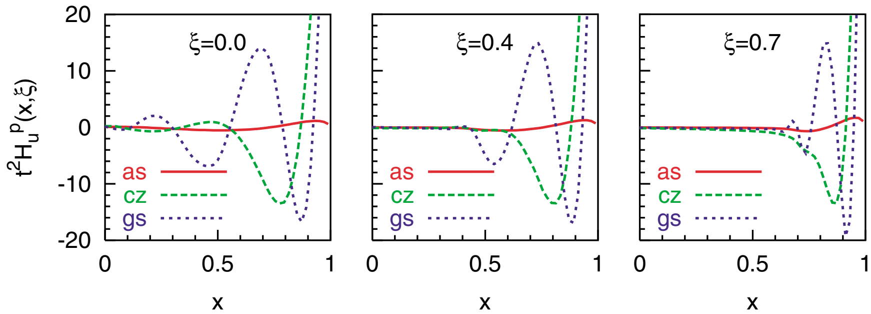

FIG. 4 (color online). $t^2 H_u(x, \xi, t)$ for the proton at $-t = 20$ GeV^2. CZ refers to the Chernyak and Zhitnitsky amplitude, GS to the Gari and Stefanis amplitude, and AS to the asymptotic amplitude.

and perturbatively calculable hard kernels. We have calculated the hard kernels for the pion and the nucleon in the leading order in α_s. As a result, data on the GPDs in the large-t regime provides a way to constrain the functional form of the distribution amplitudes.

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Note added.—After this Letter was finished, we learned that the pion case has been studied in Ref. [21]. Studying GPD in the large-t limit was first done in [22] for $\gamma^*\gamma \to \pi\pi$ [23]. We thank M. Diehl for pointing this out to us.

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