QUARK ANTISYMMETRIZATION AS A MECHANISM FOR INCREASED LENGTH SCALES IN NUCLEI*

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Talk presented at:
"NUCLEAR AND PARTICLE PHYSICS ON THE LIGHT CONE"
Los Alamos National Laboratory
July 18 – 22, 1988

Typeset in $\TeX$ by Roger L. Gilson

CTP# 1618 July 1988

* This work is supported in part by funds provided by NSF grant #8811939-INT and PSF grant C-QU/PHY(57).

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ABSTRACT

The requirement of antisymmetrizing quarks belonging to different nucleons in a nucleus has important implications for the physics of nuclei. First, it leads to a shift towards smaller values in the distribution of quark momentum inside a nucleus, relative to the distribution in isolated nucleons. Secondly, antisymmetrization modifies the quark-quark correlation, which can be interpreted as an increase in the effective size of the nucleon. Extended length scales apparently observed in nuclei may therefore be explained in good part by quark statistics without invoking a dynamical mechanism.

While it is conventional to view the nucleus as a bound state of nucleons, and to further view each nucleon as a bound state of three quarks, the requirement that the nuclear state be antisymmetrized with respect to the exchange of quarks belonging to different nucleons is not imposed in standard treatments.

Restoring the Pauli principle at the quark level for understanding certain aspects of nuclear phenomenology may be important. Sometime ago, in collaboration with R. L. Jaffe, it was shown that both the sign, as well as rough magnitude, of the EMC effect can be understood as arising from the exchange of quarks belonging to different nucleons.[1] The same physical model was also used to calculate the charge distribution as measured in elastic electron scattering.[2] Subsequently, this work was extended in two important ways. First, instead of considering a trinucleon system, nuclei with an arbitrary number of nucleons was treated. And, secondly, the effect of quark antisymmetrization on the quark-quark correlation has been calculated. This talk is a brief summary of the results obtained in Refs. [3] and [4].

Both deep inelastic electron scattering and quasi-elastic reactions seem to suggest that the length scale in nuclei is larger than the corresponding scale for nucleons. One suspects that this increase must ultimately derive from the partial deconfinement of quarks and gluons. This is also the key physical input into a QCD based explanation of the EMC effect developed by Close, Jaffe, Roberts and Ross.[5] But this apparent increase can, as we show, also be accounted for by the rather mundane necessity of antisymmetrizing the total nuclear wave function expressed in terms of quarks.

The problem of handling the combinatorics of antisymmetry is made easier by the use of second quantization and Wick's theorem. The starting point is a totally antisymmetric nuclear wavefunction.

$$|A\rangle = \frac{1}{\sqrt{N!}} \Phi^{\alpha_1 \cdots \alpha_N} A^\dagger_{\alpha_1} \ldots A^\dagger_{\alpha_N} |0\rangle \tag{1}$$

where the nucleon creation operators are three-quark composites $A^\dagger \sim q^\dagger q^\dagger q^\dagger$ where $q^\dagger$ creates a quark with a given momentum, flavor, spin component and color. Since quarks are elementary fermions, we have $\{q, q^\dagger\} \sim \delta$. But this forces upon us the conclusion that composite nucleons are not fermions because $\{A, A^\dagger\}$ is then not a delta function. The wavefunction Φ determines the motion of the three quark clusters and is an input into the problem. From Eq. (1), one can calculate the matrix elements of any one quark or two quark operator. This is a matter of combinatorics, and the result for a one quark operator is given in Ref. [3], while those for a two quark operator is in Ref. [4]. A diagrammatic interpretation allows insight into the different terms.

We mention a problem and its subsequent resolution: *a priori* it might seem that the nucleon size is a natural expansion parameter and one can straightforwardly interpolate between point nucleons and extended ones. But it turns out that

$$\langle A|A\rangle = 1 + c_1 k_F^3 r_0^3 N + c_2 k_F^6 r_0^6 N^2 + \ldots \tag{2}$$

where r_0 = nucleon size, k_F = nucleon fermi momentum, N = number of nucleons, and c_1 and c_2 are finite constants. Clearly, this implies a difficulty for large N. Fortunately, the same problem also occurs in calculating $\langle A|\hat{O}|A\rangle$ for any operator $\hat{O}$ and, by judicious arrangement of terms, one can arrange for a well-behaved value for $\langle A|\hat{O}|A\rangle / \langle A|A\rangle$. In the diagrammatic language developed in Refs. [3–4], this corresponds to cancellation of unlinked diagrams. This matter is far from trivial, but has been worked out order by order in the nuclear density.

QUARK MOMENTUM DISTRIBUTION

Consider N nucleons confined in volume V with $N \to \infty$, $V \to \infty$, but $\rho = N/V$ finite. Each nucleon is assumed to have three quarks distributed in a Gaussian wavefunction with mean square radius r_0. For simplicity, assume that there is no Fermi motion of nucleons. A pure non-interacting Fermi gas, and then a gas with a phenomenological short-range correlation, will be considered. We ask the question: What is the momentum distribution of quarks inside the nuclear environment? The answer, after considerable development, is surprisingly simple:

$$K(p) = K_{\rm dir}(p) + K_{\rm exch}(p) \tag{3}$$

$$K_{\rm dir}(p) = \left(\frac{3r_0^2}{2\pi}\right)^{3/2} e^{-\frac{3}{2}r_0^2 p^2} \tag{4}$$

$$K_{\rm exch}(p) = \left(\frac{3}{\pi}\right)^{1/2} \frac{k_F^2 r_0^2}{6} \times \left[\frac{2}{3}\left(\frac{12 r_0^2}{7\pi}\right)^{3/2} e^{-\frac{12}{7}r_0^2 p^2} + \frac{1}{3}\left(\frac{3r_0^2}{\pi}\right)^{3/2} e^{-3r_0^2 p^2} - \left(\frac{3r_0^2}{2\pi}\right)^{3/2} e^{-\frac{3}{2}r_0^2 p^2}\right] \tag{5}$$

This result, which applies to a pure isospin averaged Fermi gas of nucleons, is very pleasing. Were quark exchange absent, one would have only the "direct" term which gives the momentum distribution in an isolated nucleon. Observe that $\int d^3p\, K_{\rm dir}(p) = 1$ and $\int d^3p\, K_{\rm exch}(p) = 0$. Now note the following fact: in Eq. (5) the first two exponents (12/7,3) are bigger than the third exponent (3/2). Hence $K_{\rm exch}(p)$ is positive for small p and negative for large p. *This means that the quark momentum distribution is softened as a consequence of nucleon overlap.* Various explanations of the EMC effect also suggest this. Close *et al.* have also observed this fact, and make the connection between the model described here with other explanations.[6]

QUARK-QUARK CORRELATIONS

Let us ask for the expectation value of the following operator

$$\hat{g}(\vec{r}_1, \vec{r}_2) = \frac{1}{3N-1} \sum_{i<j} \delta(\vec{r}_1 - \vec{r}_i)\delta(\vec{r}_2 - \vec{r}_j) \tag{6}$$

where the position vectors are those of quarks. Again, we use the same nuclear model as described in the previous section. In translationally invariant matter, $\hat{g}$ can only depend on $r = |\vec{r}_1 - \vec{r}_2|$. Both direct terms, as well as those in which quarks are exchanged between nucleons, contribute. After considerable calculation, we find that, for equal numbers of neutrons and protons,

$$g(r) = \frac{3N}{V} - \frac{3N}{V}(1 - 9\epsilon)\left[\frac{3J_1(k_F r)}{k_F r}\right]^2 + 2\left(2\pi r_{\rm eff}^2\right)^{-3/2} e^{-1/2 \;\; (r^2/r_{\rm eff}^2)} \tag{7}$$

where

$$r_{\rm eff}^2 = \left(1 - \frac{99}{4}\epsilon\right) r_0^2 \tag{8}$$

$$\epsilon = -\left(\frac{3}{\pi}\right)^{1/2} \frac{k_F^3 r_0^3}{54} \; . \tag{9}$$

In Eq. (7), $J_1(k_F r)$ is a Bessel function like quantity and the notation is chosen to remind one of the usual Fermi hole. The remarkable thing is that one has a form very similar to that of a pure Fermi gas, provided that the nucleon radius is replaced by an effective radius whose value increases with density. Thus, quark exchange predicts swelling of the nucleon in a nuclear environment. Putting in a phenomenological two-body correlation, which serves to keep the nucleons apart, reduces the swelling by about 30%. This corresponds well to our intuition since quark exchange is thereby lessened.

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