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Quark-exchange contribution to the European Muon Collaboration effect in nuclear matter

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Fermi statistics require that all quarks in a nucleus be antisymmetrized, and hence that quarks belonging to different nucleons be exchanged in proportion to the degree of nucleon overlap. This leads to a shift in the distribution of quark momentum relative to that in isolated nucleons, and hence to a shift in the structure function $F_2(x)$. This paper extends previous work on the European Muon Collaboration (EMC) effect for the trinucleon system to systems with large numbers of nucleons. For large N, certain divergences were encountered and traced to contributions arising from unlinked quark clusters. After renormalization a smooth $N\rightarrow\infty$ limit was obtained. The conclusion derived in the previously considered $N=3$ case is upheld: Quark exchange is very important—perhaps even dominant—in calculating the deviation away from unity of the ratio of the nuclear structure function to that of a free nucleon.

I. INTRODUCTION

Quark exchange contributions to the nuclear structure function $F_2(x)$ arise because quarks in different nucleons must be antisymmetrized. This was demonstrated by Jaffe[1] in a simple, solvable one-dimensional model. A realistic calculation with three-dimensional Fadeev nuclear wave functions for $N=3$ nuclei was subsequently undertaken by Hoodbhoy and Jaffe.[2] The results of this calculation were surprising: The exchange of quarks between the three nucleons led to a shift in the nuclear structure function which was almost of the same magnitude and shape as the expected European Muon Collaboration (EMC) ratio.

The three nucleon system was chosen in Ref. 2 for good reasons: It is fairly dense, has reliable wave functions directly obtained from solving the Schrödinger equation with realistic nuclear forces, and a calculation appeared more tractable for smaller N. But, on the other hand, this was a compromise solution because there is no deep inelastic lepton scattering data from trinucleon systems. Furthermore, the complexity of the three nucleon wave function somewhat obscured the simplicity of the physical ideas. And finally, the connection with heavier nuclei was far from evident.

In this paper we have undertaken the task of extending the treatment of Ref. 2 by considering nuclei with an arbitrary number of nucleons N. More specifically, we seek to evaluate the quark exchange contribution to the EMC effect in nuclear matter. This involves essentially two steps: One must first calculate the shift in quark momentum distribution originating from quark exchanges, and then calculate the effect of this on the nuclear structure function. The latter has been fully dealt with in Ref. 2 and will not be repeated. However, the former step involves a new element; in the $N\rightarrow\infty$ limit, quark exchange between nucleons leads to the occurrence of certain divergences and hence the relevant matrix elements must be appropriately renormalized. In the following paragraphs we briefly explain this point as well as summarize the essential physics underlying the calculation.

Conventional nuclear physics, viewed in the second quantization formalism, takes on a very simple appearance. If $a_\alpha^\dagger$ and a_β denote the creation and destruction operators of an elementary nucleon, then one takes $\{a_\alpha^\dagger,a_\beta\}=\delta_{\alpha\beta}$ and all other anticommutators to be zero. The most general nuclear state with N nucleons is

$$\frac{1}{\sqrt{N!}}\Phi^{\alpha_1\alpha_2\cdots\alpha_N}a_{\alpha_1}^\dagger a_{\alpha_2}^\dagger\cdots a_{\alpha_N}^\dagger|0\rangle ,$$

where a summation on repeated indices is implied, and each index refers to a complete set of nucleon quantum numbers (e.g., momentum, spin, and isospin). The wave function Φ is totally antisymmetric and couples N nucleons to give the total nuclear momentum, spin, and isospin. The Hartree Fock approximation amounts to taking Φ as a determinant of single nucleon wave functions, and so on.

The above picture must certainly become invalid at some level if nucleons are not fundamental particles but, instead, are quark composities. The QCD bound state problem is intractable. However, as a first step towards including quarks in nuclei, one can envisage making the replacement $a^\dagger\rightarrow A^\dagger$ where $A^\dagger$ creates *a physical,* extended nucleon rather than an elementary point nucleon. The nuclear state—which is the basic ansatz made here—is then assumed to be

$$|A\rangle=\frac{1}{\sqrt{N!}}\Phi^{\alpha_1\alpha_2\cdots\alpha_N}A_{\alpha_1}^\dagger A_{\alpha_2}^\dagger\cdots A_{\alpha_N}^\dagger|0\rangle . \qquad (1.1)$$

Again, Φ is totally antisymmetric but it now describes the motions of the center of mass of the N physical nucleons. If a physical nucleon is assumed to be constituted of three valence quarks only, then, symbolically, $A^\dagger\sim q^\dagger q^\dagger q^\dagger$ where $q^\dagger$ creates a quark with a given momentum, flavor, spin component, and color. Since quarks are elementary fermions, one has $\{q^\dagger,q\}\sim\delta$. But this immediately forces upon us the conclusion that $\{A_\alpha^\dagger,A_\beta\}\neq\delta_{\alpha\beta}$. The reader may readily verify the correct relation or look it up in Ref. 2. In any system with finite nucleon density, there will be correction terms to the anticommutator which physically correspond to the exchange of quarks between nucleons.

To obtain the momentum distribution of quarks in this model, one needs to calculate an expectation value of the kind

$$K_{\mu\nu}=\frac{\langle A \mid q_\mu^\dagger q_\nu \mid A \rangle}{\langle A \mid A \rangle} \tag{1.2}$$

in terms of the nuclear wave function Φ, defined by Eq. (1.1), and the nucleon wave function, to be defined in Eq. (2.1).

A priori it might seem that the nucleon size b is a natural expansion parameter and one can straightforwardly interpolate between point nucleons and extended ones. But it will be shown that,

$$\langle A \mid A \rangle = 1+O(b^3K_F^3N)+O(b^6K_F^6N^2)+\cdots \ . \tag{1.3}$$

Here K_F is a length scale in momentum space. For any $b\neq 0$, the expression is badly divergent for large N. The numerators in Eq. (1.2) are also similarly divergent and the quotient is ill defined. At this point one is reminded of a certain situation in the usual many body theory: The overlap $\langle \Phi \mid \Psi \rangle$ of any approximate state $\mid \Phi \rangle$ with the exact state $\mid \Psi \rangle$ is exactly zero in the $N \to \infty$ limit irrespective of how good $\mid \Phi \rangle$ may be. Nevertheless, the expectation value of few body operators can be calculated perfectly well in the state $\mid \Phi \rangle$. This is possible because a massive cancellation occurs between disconnected diagrams in the numerator against corresponding diagrams in the denominator, this being the essence of various linked cluster theorems.[3] The disease which appears to afflict (1.2) and (1.3) will also be traced to the existence of unlinked diagrams, although here a diagram shall correspond to a particular symmetry arrangement of quarks rather than a rule for dynamical calculations.

II. FORMULATION

The notation used in the following is that of Ref. 2. The symbol α shall denote the coordinates of a nucleon $\{\mathbf{K},M_S,M_T\}$, or any other complete set. Similarly μ,ν,ρ,σ are quark coordinates, $\{\mathbf{k},m_s,m_t,c\}$. The repeated index summation convention is constantly assumed, and it implies integration in the event of a continuous index. The nucleon wave function $C^\alpha_{\mu_1\mu_2\mu_3}$ is defined by

$$\mid \alpha \rangle = A_\alpha^\dagger \mid 0 \rangle = \frac{1}{\sqrt{3!}} C^\alpha_{\mu_1\mu_2\mu_3} q^\dagger_{\mu_1} q^\dagger_{\mu_2} q^\dagger_{\mu_3} \mid 0 \rangle \ , \tag{2.1}$$

and the normalization $\langle \alpha \mid \alpha' \rangle = \delta^{\alpha\alpha'}$ implies that

$$C^\alpha_{\mu_1\mu_2\mu_3} C^{\alpha'}_{\mu_1\mu_2\mu_3} = \delta^{\alpha\alpha'} \ . \tag{2.2}$$

In this section the precise form of C is irrelevant, other than its spatial extension should correspond to the dimensions of a nucleon.

Consider a matrix element of the following kind:

$$\langle 0 \mid (q_{\nu_3} q_{\nu_2} q_{\nu_1})(\ \ ) \cdots (\ \ )(q^\dagger_{\mu_1} q^\dagger_{\mu_2} q^\dagger_{\mu_3}) \mid 0 \rangle \ . \tag{2.3}$$

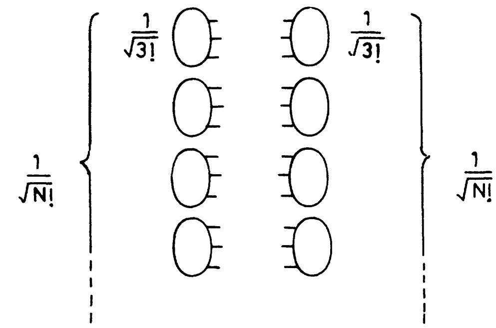

FIG. 1. A diagrammatic representation of $\langle A \mid A \rangle$. All hooks on the left must be connected to all hooks on the right in all possible ways.

Wick's theorem states that this is given by the sum of products of all contractions. For efficient bookkeeping of the vast number of terms, let us define a diagrammatic representation in the following way. Draw two sets of N blobs, as in Fig. 1, with each blob having three hooks. A numerical factor of $1/\sqrt{3!}$ is associated with each blob, and $1/\sqrt{N!}$ with each set of N blobs.

Exchanging any two blobs on the same side, or any two hooks in a blob, leads to a sign change. Wick's theorem amounts to the statement that all hooks on the left be joined by lines to all hooks on the right in all possible ways. It is easy to see that an even (odd) number of line crossings gives a net positive (negative) sign, respectively. The vast number of possibilities can obviously be reduced to a much smaller number of topologically distinct diagrams. Two diagrams will be said to be equivalent if, by adhering to the above rules, they can be made to yield the same mathematical expression.

For large N the number of distinct diagrams is hopelessly large, and for practical purposes we make the following approximation: All those diagrams shall be discarded which involve simultaneous quark exchanges between three, or more than three, nucleons. Examples are given in Fig. 2. This is justified if either the nucleon size is small or the density of nucleons is sufficiently low. However, we must include quark exchange between any number of nucleon pairs which are dynamically uncorrelated from other pairs. An example is given in Fig. 3.

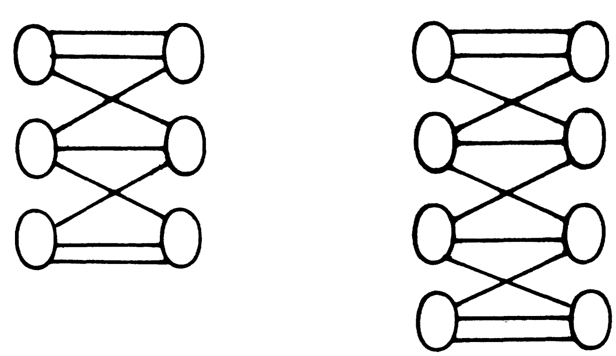

FIG. 2. Examples of diagrams which must be discarded in the two nucleon overlap approximation.

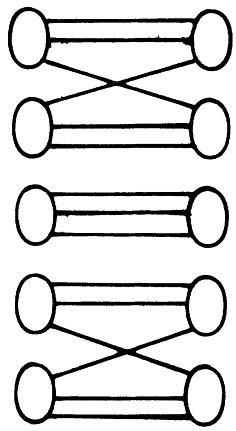

FIG. 3. Two interacting pairs of nucleons. Such diagrams must be included.

Although the total number of diagrams which enter into the calculation of various matrix elements is extremely large, they are repetitions of a few basic units, or "primitives," only. In the following a complete catalogue is presented.

(a) Primitives entering into $\langle A \mid A \rangle$ are illustrated in Fig. 4. The corresponding expressions are

$$\delta(\alpha_1\alpha_1')=C^{\alpha_1}_{\mu_1\mu_2\mu_3}C^{\alpha_1'}_{\mu_1\mu_2\mu_3}\ , \tag{2.4}$$

$$L(\alpha_1\alpha_2,\alpha_1'\alpha_2')=C^{\alpha_1}_{\mu_1\mu_2\mu_3}C^{\alpha_1'}_{\mu_1\mu_2\rho_3}C^{\alpha_2}_{\rho_1\rho_2\rho_3}C^{\alpha_2'}_{\rho_1\rho_2\mu_3}\ . \tag{2.5}$$

The correspondence between Eq. (2.5) and Fig. 4(b) should be self explantory: The hooks of the left upper blob have been labeled as μ's while those on the one below it are ρ's. The ordering of indices reflects the crossing of a line from 1 to 2′ and of another from 2 to 1′.

(b) Primitives entering into $\langle A \mid q^\dagger_\mu q_\nu \mid A \rangle$ are illustrated in Fig. 5. The corresponding expressions are

$$l_{\mu\nu}(\alpha,\alpha')=C^{\alpha}_{\mu\mu_2\mu_3}C^{\alpha'}_{\nu\mu_2\mu_3}\ , \tag{2.6}$$

$$\mathcal{M}_{\mu\nu}(\alpha_1\alpha_2,\alpha_1'\alpha_2')=C^{\alpha_1}_{\mu\mu_2\mu_3}C^{\alpha_1'}_{\nu\mu_2\rho_3}C^{\alpha_2}_{\rho_1\rho_2\rho_3}C^{\alpha_2'}_{\rho_1\rho_2\mu_3}\ , \tag{2.7}$$

$$\mathcal{N}_{\mu\nu}(\alpha_1\alpha_2,\alpha_1'\alpha_2')=C^{\alpha_1}_{\mu\mu_2\mu_3}C^{\alpha_1'}_{\mu_2\mu_3\rho_3}C^{\alpha_2}_{\rho_1\rho_2\rho_3}C^{\alpha_2'}_{\rho_1\rho_2\nu}\ . \tag{2.8}$$

For brevity the dependence of (2.5)–(2.8) on the nucleon coordinates α will be suppressed in the forthcoming. Since all internal quark lines have been summed over, they do not appear on the left. If we also sum over external quark indices, the following useful relations are obtained:

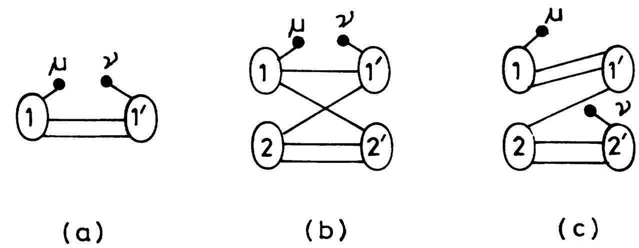

FIG. 5. All primitives for $\langle A \mid q^\dagger_\mu q_\nu \mid A \rangle$.

$$l_{\mu\mu}=1\ , \tag{2.9a}$$

$$\mathcal{M}_{\mu\mu}=\mathcal{N}_{\mu\mu}=L\ . \tag{2.9b}$$

The above follow from Figs. 5 since a contraction of external indices corresponds to joining them with a line. In conventional nuclear physics, where quarks are not antisymmetrized between separate nucleons, $L=\mathcal{M}=\mathcal{N}=0$.

Having identified all the necessary ingredients, let us proceed to calculate the matrix elements of interest. First, consider $\langle A \mid A \rangle$. Its value is given by the sum of a series of terms with $r=0,1,2,\ldots$ two nucleon clusters as in Fig. 6. The result is

$$\langle A \mid A \rangle=\sum_{r=0}(-1)^r\frac{N!}{r!(N-2r)!}\left[\frac{9}{2}\right]^r\langle L^r\rangle\ , \tag{2.10}$$

where L^r is $LL\cdots L$ r times and $\langle\ \rangle$ denotes averaging of the enclosing operator in the nucleon Hilbert space with respect to the appropriate nuclear density matrix $\rho^{(n)}$ constructed from Φ,

$$\rho^{(n)}(\alpha_1\cdots\alpha_n,\alpha_1'\cdots\alpha_n')=\frac{1}{N!}\Phi^{*\alpha_1\cdots\alpha_n\alpha_{n+1}\cdots\alpha_N}\Phi^{\alpha_1'\cdots\alpha_n'\alpha_{n+1}\cdots\alpha_N}\ . \tag{2.11}$$

As an illustration

$$\langle L^2\rangle=L(\alpha_1\alpha_2,\alpha_1'\alpha_2')L(\alpha_3\alpha_4,\alpha_3'\alpha_4')\times\rho^{(4)}(\alpha_1\alpha_2\alpha_3\alpha_4,\alpha_1'\alpha_2'\alpha_3'\alpha_4')\ . \tag{2.12}$$

The normalization of Φ has been chosen so that

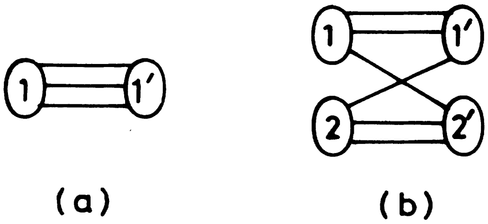

FIG. 4. All primitives for $\langle A \mid A \rangle$.

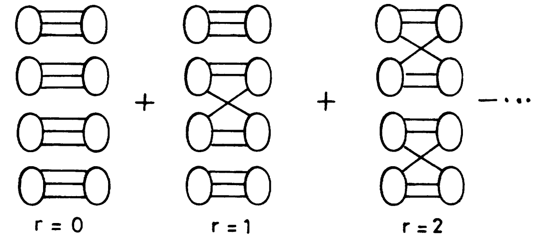

FIG. 6. Contributions to $\langle A \mid A \rangle$.

$\rho^{(0)}=\langle 1\rangle=1$.

We now justify Eq. (2.10). For any diagram containing r clusters, such as in Fig. 6, all except r indices will be common to both Φ^* and Φ. Hence only $\rho^{(2r)}$ occurs, contracted with a product of r L's. The factor $N!/(2!)^r(N-2r)!$ is the number of ways of choosing r pairs from N nucleons, and must be divided by $r!$ because all pairs are equivalent. The factor 9 accompanying each L arises from counting the number of different ways of connecting two blobs on the left with two blobs on the right, and then dividing by the normalization. Finally, $(-1)^r$ comes from the rule that even (odd) numbers of quark line crossings give a net positive (negative) result, respectively.

Next, consider $\langle A \mid q_\mu^\dagger q_\nu \mid A\rangle$. Diagrammatically, its evaluation comes from first connecting the two sets of N blobs in all possible ways, and then breaking a single line and naming the ends as μ and ν. Suppressing, as before, explicit reference to nucleon coordinates, the result is the following:

$$\langle A \mid q_\mu^\dagger q_\nu \mid A\rangle = 3\sum_{r=0}(-1)^r\frac{N!}{r!(N-2r-1)!}\left[\frac{9}{2}\right]^r\langle l_{\mu\nu}L^r\rangle + \sum_{r=1}(-1)^r\frac{N!}{(r-1)!(N-2r)!}\left[\frac{9}{2}\right]^r\langle W_{\mu\nu}L^{r-1}\rangle\,, \tag{2.13}$$

where W is, in terms of the primitives $\mathcal{M}$ and $\mathcal{N}$,

$$W_{\mu\nu}=4\mathcal{M}_{\mu\nu}+2\mathcal{N}_{\mu\nu}\,. \tag{2.14}$$

The occurence of $(N-2r-1)!$ instead of $(N-2r)!$ in the first term of (2.14) is a reminder that one nucleon has been singled out by the external quark indices. Also, in the second term the $(r-1)!$ occurs because there are only $(r-1)$ indistinguishable pairs, the pair associated with $\mathcal{M}$ or $\mathcal{N}$ being singled out.

To check the correctness of (2.14), set $\mu=\nu$ and sum on μ. Using Eqs. (2.9) and (2.14),

$$\begin{aligned}\langle A \mid q_\mu^\dagger q_\nu \mid A\rangle &= \sum_r(-1)^r\frac{N!}{(r-1)!(N-2r-1)!}\left[\frac{3}{r}+\frac{6}{N-2r}\right]\left[\frac{9}{2}\right]^r\langle L^r\rangle\\ &=3N\sum_r(-1)^r\frac{N!}{r!(N-2r)!}\left[\frac{9}{2}\right]^r\langle L^r\rangle\\ &=3N\langle A \mid A\rangle\,.\end{aligned} \tag{2.15}$$

This is, indeed, as it should be. Had our approximation not respected this basic relation, spurious results would have been expected for the nuclear structure function because its moments would have contained the wrong number of valence quarks in the target.

III. RENORMALIZATION

We shall now confront the serious difficulty referred to in the Introduction: The matrix elements calculated so far are all badly divergent when the number of nucleons N becomes large.

To see the source of the problem with greater clarity, consider for the moment a hypothetical system of N spinless, single species, "nucleons" each composed of three spinless and flavorless quarks. The nuclear wave function Φ is assumed to be the usual plane wave Slater determinant, corresponding to the free motion of nucleons inside a volume V. There are no dynamical correlations coming from hardcore repulsion, although Pauli correlations are certainly present. The two body nucleon density matrix is

$$\rho^{(2)}(\mathbf{K}_1\mathbf{K}_2,\mathbf{K}_1'\mathbf{K}_2') = \frac{1}{N(N-1)}[\delta(\mathbf{K}_1-\mathbf{K}_1')\delta(\mathbf{K}_2-\mathbf{K}_2') - \delta(\mathbf{K}_1-\mathbf{K}_2')\delta(\mathbf{K}_2-\mathbf{K}_1')]\,. \tag{3.1}$$

All nucleon momenta are bounded in magnitude by the Fermi momentum K_F, where $K_F^3=6\pi^2N/V$. In addition to this nuclear model, we shall also require a model for the nucleon which shows it to be an extended object which can overlap with other nucleons. As in Eq. (2.4) of Ref. 2, $C^\alpha_{\mu_1\mu_2\mu_3}$ will be taken to be a product of Gaussians and, for the present only, μ_i is a quark momentum. Performing the integrations in Eq. (2.5) gives

$$L(\mathbf{K}_1\mathbf{K}_2,\mathbf{K}_1'\mathbf{K}_2') = \delta(\mathbf{K}_1+\mathbf{K}_2-\mathbf{K}_1'-\mathbf{K}_2')\left[\frac{3b^2}{4\pi}\right]^{3/2} \times\exp\left[-\frac{b^2}{12}(\mathbf{K}-\mathbf{K}')^2\right]\exp\left[-\frac{b^2}{12}\left[\frac{\mathbf{K}+\mathbf{K}'}{2}\right]\right]^2, \tag{3.2}$$

where $\mathbf{K}=\mathbf{K}_1-\mathbf{K}_2$ and $\mathbf{K}'=\mathbf{K}_1'-\mathbf{K}_2'$ are the relative momenta of initial and final nucleons, and b is the nucleon rms radius. The quartity $\langle L\rangle$ may now be calculated from Eqs. (3.1) and (3.2),

$$\langle L\rangle=\int d\mathbf{K}_1 d\mathbf{K}_2 d\mathbf{K}_1' d\mathbf{K}_2' \rho^{(2)}(\mathbf{K}_1\mathbf{K}_2,\mathbf{K}_1'\mathbf{K}_2') \times L(\mathbf{K}_1\mathbf{K}_2,\mathbf{K}_1'\mathbf{K}_2')\,. \tag{3.3}$$

Although the indicated integrations can be carried out exactly, the answer is in terms of error functions because the upper limit of the integrations K_F is finite. It is more revealing to instead expand the exponentials in (3.2) to give a series in increasing powers of density,

$$N\langle L\rangle=\frac{3}{20}\left[\frac{3}{\pi}\right]^{1/2}b^5K_F^5+\cdots\ . \tag{3.4}$$

The implication of Eq. (3.4) for the normalization $\langle A\mid A\rangle$ is clear. From Eq. (2.10),

$$\begin{aligned}\langle A\mid A\rangle&=1-\tfrac{9}{2}N(N-1)\langle L\rangle\\&\quad+\frac{1}{2!}\left[\frac{9}{2}\right]^2N(N-1)(N-2)(N-3)\langle L\rangle^2+\cdots\\&=1+O(N)+O(N^2)+\cdots\ .\end{aligned} \tag{3.5}$$

This divergence can be understood physically as well—as long as $b\neq 0$, the number of single pairs which contribute to $\langle A\mid A\rangle$ grows as $\frac{1}{2}N(N-1)$, the number of double pairs as $\frac{1}{4}N(N-1)(N-2)(N-3)$, and so on. A similar problem besets $\langle A\mid\hat{O}\mid A\rangle$ for any quark operator $\hat{O}$. But, of course, neither matrix element is separately observable, only the quotient is. Hence, one must endeavor to somehow cancel the bad behavior of one against that of the other.

To proceed further, it will be necessary to know how to take the expectation value of 3,4,5,... nucleon operators in the nuclear ground state wave function. By explicitly writing down the density matrices in the Fermi gas model or otherwise, it can be readily checked that for the diagonal matrix elements ($\mu=\nu$),

$$\langle l_{\mu\nu}L^r\rangle=\langle l_{\mu\nu}\rangle\langle L^r\rangle\ . \tag{3.6}$$

This is true because l is diagonal in the nucleon space if it is diagonal in the quark space [see Eq. (2.6)]. Using Eqs. (2.10), (2.13), and (3.6),

$$K_{\mu\nu}=\frac{3\langle l_{\mu\nu}\rangle\sum_{r=0}(-1)^r\frac{N!}{r!(N-2r-1)!}\left[\frac{9}{2}\right]^r\langle L^r\rangle+\sum_{r=1}(-1)^r\frac{N!}{(r-1)!(N-2r)!}\left[\frac{9}{2}\right]^r\langle W_{\mu\nu}L^{r-1}\rangle}{1+\sum_{r=1}(-1)^r\frac{N!}{r!(N-2r)!}\left[\frac{9}{2}\right]^r\langle L^r\rangle}\ . \tag{3.7}$$

Imagine now that for fixed N the value of b is chosen small enough so that the denominator can be expanded according to $(1+x)^{-1}=1-x+x^2-x^3\cdots$, where x stands for the indicated power series in $\langle L^r\rangle$ beginning from $r=1$. Now multiply the numerator of Eq. (3.7) by this expansion of its denominator and group together terms according to their power of $\langle L^r\rangle$. It is necessary to regard $l_{\mu\nu}$ as of order unity (recall, $l_{\mu\mu}=1$), and $W_{\mu\nu}$ as of order L (recall, $W_{\mu\mu}=6L$). The resulting series, which is a ratio of two strongly divergent series, is smooth and apparently convergent. Up to second order in L (two sets of quark exchanges) the result for $K_{\mu\nu}$ is

$$K_{\mu\nu}=3N(K_{\mu\nu}^{(0)}+K_{\mu\nu}^{(1)}+K_{\mu\nu}^{(2)}+\cdots)\ , \tag{3.8a}$$

where

$$K_{\mu\nu}^{(0)}=\langle l_{\mu\nu}\rangle\ , \tag{3.8b}$$

$$K_{\mu\nu}^{(1)}=9N(\langle l_{\mu\nu}\rangle\langle L\rangle-\tfrac{1}{6}\langle W_{\mu\nu}\rangle)\ , \tag{3.8c}$$

$$\begin{aligned}K_{\mu\nu}^{(2)}&=81N^2[\tfrac{1}{2}N\langle l_{\mu\nu}\rangle(\langle L\rangle^2-\langle L^2\rangle)+2\langle l_{\mu\nu}\rangle\langle L\rangle^2\\&\quad+\tfrac{1}{12}N(\langle W_{\mu\nu}L\rangle-\langle W_{\mu\nu}\rangle\langle L\rangle)\\&\quad-\tfrac{1}{3}\langle W_{\mu\nu}\rangle\langle L\rangle]\ .\end{aligned} \tag{3.8d}$$

Since $N\langle L\rangle$ is finite and small [see Eq. (3.4)] it is clear that $K_{\mu\nu}^{(1)}$ is also finite as $N\to\infty$. To see the finiteness of $K_{\mu\nu}^{(2)}$ one must use $\langle L^2\rangle=\langle L\rangle^2+O(1/N)$ and $\langle WL\rangle=\langle W\rangle\langle L\rangle+O(1/N)$. Both facts are readily established either by explicit recourse to the density matrices of the Fermi gas model, or by the observation that the difference between left- and right-hand sides is an exchange term (in nucleon space) and hence is proportional to the probability of finding one particular nucleon $(1/N)$. Clearly $K_{\mu\nu}^{(2)}$ is of second order in the small parameter $N\langle L\rangle$ and so the series is expected to be rapidly convergent. Because the algebra gets steadily more complicated, we have verified the cancellation up to fourth order only. However, a general proof should be possible. In diagrammatic terms, our procedure involves the cancellation of the infinite parts arising from unlinked quark clusters in the numerator with those in the denominator.

Finally, as a check on the algebra, observe that setting $\mu=\nu$ and summing over μ gives, after use of (2.9) and (2.14), that $K_{\mu\mu}^{(0)}=1$ and $K_{\mu\mu}^{(1)}=K_{\mu\mu}^{(2)}=\cdots=0$.

IV. CALCULATION OF DIAGRAMS

Two main inputs are needed in the present model for calculating the quark momentum distribution in a nuclear medium. First, one needs a model for the quark distribution in free nucleons. And second, at the simplest level, the two body nuclear density matrix. The latter is needed, rather than the full N body wave function, because the density of nuclear matter is assumed low enough so that only pairs of nucleons can overlap significantly.

Considering, for the moment, all indices μ,ν,ρ, etc. as referring to quark momenta only and the indices α to nucleon momenta only, integrations over internal quark momenta are performed as indicated by the expressions (2.6)–(2.8). The resulting quantities are operators in the space of nucleons:

$$l_P(\mathbf{K}_1,\mathbf{K}_1')=\delta(\mathbf{K}_1-\mathbf{K}_1')\left[\frac{3b^2}{2\pi}\right]^{3/2}\exp[-\tfrac{3}{2}b^2(\mathbf{p}-\tfrac{1}{3}\mathbf{K}_1)^2]\;, \tag{4.1}$$

$$\mathcal{M}_P(\mathbf{K}_1\mathbf{K}_2,\mathbf{K}_1'\mathbf{K}_2')=\delta(\mathcal{K}-\mathcal{K}')\left[\frac{9b^4}{28\pi^2}\right]^{3/2}\exp\left[-\frac{b^2}{12}(\mathbf{K}-\mathbf{K}')^2\right]\exp\left[\frac{b^2}{12}\left[\frac{\mathbf{K}+\mathbf{K}'}{2}\right]^2\right]$$
$$\times\exp[-\tfrac{12}{7}b^2(\mathbf{p}-\tfrac{1}{3}\mathcal{K}-\tfrac{1}{8}\mathbf{K}-\tfrac{1}{8}\mathbf{K}')^2]\;, \tag{4.2}$$

$$\mathcal{N}_P(\mathbf{K}_1\mathbf{K}_2,\mathbf{K}_1'\mathbf{K}_2')=\delta(\mathcal{K}-\mathcal{K}')\left[\frac{9b^4}{16\pi^2}\right]^{3/2}\exp\left[-\frac{b^2}{12}(\mathbf{K}-\mathbf{K}')^2\right]\exp\left[-\frac{b^2}{12}\left[\frac{\mathbf{K}+\mathbf{K}'}{2^2}\right]\right]$$
$$\times\exp[-3b^2(\mathbf{p}-\tfrac{2}{3}\mathcal{K}-\tfrac{1}{4}\mathbf{K}+\tfrac{1}{4}\mathbf{K}')^2]\;. \tag{4.3}$$

In the above, p refers to the external quark momentum ($\mu=\nu=p$). The "initial" nucleon momenta are $\mathbf{K}_1,\mathbf{K}_2$ and "final" momenta are $\mathbf{K}_1',\mathbf{K}_2'$. In terms of these $\mathbf{K}=\mathbf{K}_1-\mathbf{K}_2$ and $\mathbf{K}'=\mathbf{K}_1'-\mathbf{K}_2'$ are the relative momenta of the interacting pair, and the c.m. momenta $2\mathcal{K}=\mathbf{K}_1+\mathbf{K}_2$ and $2\mathcal{K}'=\mathbf{K}_1'+\mathbf{K}_2'$ are equal by virtue of the delta function.

The remaining quantum numbers—color, spin, isospin—can be conveniently discussed at this point. The color factor is the same for $L,\mathcal{M},\mathcal{N}$ and is

$$\left[\frac{1}{\sqrt{3!}}\right]^4\varepsilon_{\mu_1\mu_2\mu_3}\varepsilon_{\mu_1\mu_2\rho_3}\varepsilon_{\rho_1\rho_2\rho_3}\varepsilon_{\rho_1\rho_2\mu_3}=\tfrac{1}{3}\;.$$

Here the indices μ,ρ run from 1 to 3. Spin and isospin can be treated similarly but the procedure is more complicated because one must perform sums over the m quantum numbers in a product of 16 SU(2) Clebsch-Gordan coefficients. However, it is greatly simplifying to deal with symmetric nuclear matter in which all magnetic substates are occupied equally. This enables closure relations to be used. Also, it is convenient to work in the basis where the interacting pair is coupled to total spin S and total isospin T since these are conserved. We omit details of the spin-isospin sums since the principle is quite clear. The reader is referred to Ref. 2 where a similar sum was performed.

A. Fermi gas model

Using the two body density matrix in the Fermi gas model whose momentum part is given in Eq. (3.1) [but now $K_F^3=\frac{3}{2}\pi^2(N/V)$ because spin and isospin are included], it is now possible to calculate $\langle L\rangle$, $\langle l\rangle$, $\langle\mathcal{M}\rangle$, and $\langle\mathcal{N}\rangle$ without further assumptions. However, as remarked earlier, the results are in terms of error functions. It is much more illuminating, and consistent with our assumption of a dilute Fermi gas, to keep only the leading order corrections in density. Furthermore, we shall also assume that $p_{\text{quark}} \gg P_{\text{nucleon}}$, i.e., neglect Fermi motion. Again, this is purely for numerical convenience. The effect of Fermi motion on the nuclear structure function is well understood,[4] and a crude estimate of this in the present model gave qualitatively the right behavior. Hence this is not an important omission. Subject to the approximations discussed here, the results for symmetric, low density, nuclear matter are

$$\langle l_p\rangle=\left[\frac{3b^2}{2\pi}\right]^{3/2}e^{-(3/2)b^2p^2}\;, \tag{4.4}$$

$$N\langle L\rangle=-\left[\frac{3}{\pi}\right]^{1/2}\frac{\alpha^3}{54}\;, \tag{4.5}$$

$$N\langle\mathcal{M}_p\rangle=-\left[\frac{3}{\pi}\right]^{1/2}\frac{\alpha^3}{54}\left[\frac{12b^2}{7\pi}\right]^{3/2}e^{-(12/7)b^2p^2}\;, \tag{4.6}$$

$$N\langle\mathcal{N}_p\rangle=-\left[\frac{3}{\pi}\right]^{1/2}\frac{\alpha^3}{54}\left[\frac{3b^2}{\pi}\right]^{3/2}e^{-3b^2p^2}\;. \tag{4.7}$$

The dimensionless parameter $\alpha=bK_F$ determines the strength of the exchange corrections, and is of order one for a typical nucleon radius b and Fermi momentum K_F.

Inserting the above into Eq. (3.8), and defining $K(p)d^3p$ as the probability of finding a quark in the momentum range d^3p, we obtain

$$K(p)=K_{\text{dir}}(p)+K_{\text{exch}}(p)\;, \tag{4.8a}$$

where the direct part is simply that of an isoscalar and spin-averaged nucleon at rest and the exchange part is the sought after density dependent piece,

$$K_{\text{dir}}(p)=\left[\frac{3b^2}{2\pi}\right]^{3/2}e^{-(3/2)b^2p^2}\;, \tag{4.8b}$$

$$K_{\text{exch}}(p)=\left[\frac{3}{\pi}\right]^{1/2}\frac{\alpha^3}{6}\left[\frac{2}{3}\left[\frac{12b^2}{7\pi}\right]^{3/2}e^{-(12/7)b^2p^2}+\frac{1}{3}\left[\frac{3b^2}{\pi}\right]^{3/2}e^{-3b^2p^2}-\left[\frac{3b^2}{2\pi}\right]^{3/2}e^{-(3/2)b^2p^2}\right]\;. \tag{4.8c}$$

This is a very simple and pleasing result in spite of the fact that it was obtained in the pure Fermi gas model in which nucleons are totally free, except for Pauli correlations, to roam over the entire nuclear volume V. Observe that $\int d^3pK_{\text{dir}}(p)=1$ and $\int d^3pK_{\text{exch}}(p)=0$. The second relation, together with the fact that the first two exponents $(\frac{12}{7},3)$ are bigger than the third exponent $(\frac{3}{2})$ in Eq. (4.8c), implies that $K_{\text{exch}}(p)$ is positive for small p and negative for large p. *This means that the quark momentum distribution in nuclear matter is softened as a consequence of nucleon overlap.* This is precisely what some other explanations of the EMC effect also suggest.[5]

B. Correlations

At this point one can justifiably object that results obtained in a free Fermi gas have doubtful quantitative value because quark exchange is intrinsically a short range process. This, in fact, was one of the chief reasons for considering the trinucleon system in Ref. 2. In spite of the considerable complications coming from the use of realistic wave functions. Here we shall introduce hardcore correlations between the interacting nucleon pair in a manner which is somewhat *ad hoc*, but which nevertheless has a good intuitive basis. Specifically, we obtain the two body density matrix in coordinate space from the corresponding Fermi gas expression,

$$\rho^{(2)}(\mathbf{R}_1\mathbf{R}_2,\mathbf{R}_1'\mathbf{R}_2') =[1-f(|\mathbf{R}_1-\mathbf{R}_2|)]\rho_{\text{FG}}^{(2)}(\mathbf{R}_1\mathbf{R}_2,\mathbf{R}_1'\mathbf{R}_2') \times[1-f(|\mathbf{R}_1'-\mathbf{R}_2'|)]\ , \tag{4.9}$$

where the correlation function $f(R)$ has limits $f(0)=1$ and $f(\infty)=0$. The effect of the factor 1-f is to push nucleons apart. A convenient choice for $f(R)$ is given by Miller and Spencer,[6]

$$f(R)=e^{-\gamma^2R^2}(1-\beta^2R^2)\ . \tag{4.10}$$

The parameters β,γ determine the "wound" parameter[6] of the Brueckner-Bethe treatment of nuclear matter.

Correlations are included in the quark exchange corrections by obtaining the momentum space density matrix from (4.9) by Fourier transformation, and then recalculating $\langle L\rangle$, $\langle \mathcal{M}\rangle$, and $\langle \mathcal{N}\rangle$. The result is strikingly simple: $\langle L\rangle\to U\langle L\rangle$, $\langle \mathcal{M}\rangle\to U\langle \mathcal{M}\rangle$, and $\langle \mathcal{N}\rangle\to U\langle \mathcal{N}\rangle$, where

$$U=1-2\left[1+\beta^2\frac{\partial}{\partial\gamma^2}\right](1+\tfrac{5}{3}\alpha^2\gamma^2)^{-3/2} +\left[1+\beta^2\frac{\partial}{\partial\gamma_1^2}\right]\left[1+\beta^2\frac{\partial}{\partial\gamma_2^2}\right] \times[1+\tfrac{5}{3}\alpha^2(\gamma_1^2+\gamma_2^2)+\tfrac{16}{9}\alpha^4\gamma_1^2\gamma_2^2]^{-3/2}\ , \tag{4.11}$$

The derivatives are to be evaluated at the point $\gamma_1=\gamma_2=\gamma$. Typically, U is about 0.5 showing that correlations decrease the quark exchange corrections by as much as 50%.

Plotted in Fig. 7 is the Gaussian momentum distribution $4\pi p^2K_{\text{dir}}(p)$ for a stationary, spin and isospin averaged nucleon with rms radius $b=0.8$ fm. Shown on a magnified scale is the quark exchange correction $4\pi p^2K_{\text{exch}}(p)$ for a Fermi gas ($\gamma=\infty$), and for correlated nuclear matter with realistic parameters[6] for β,γ.

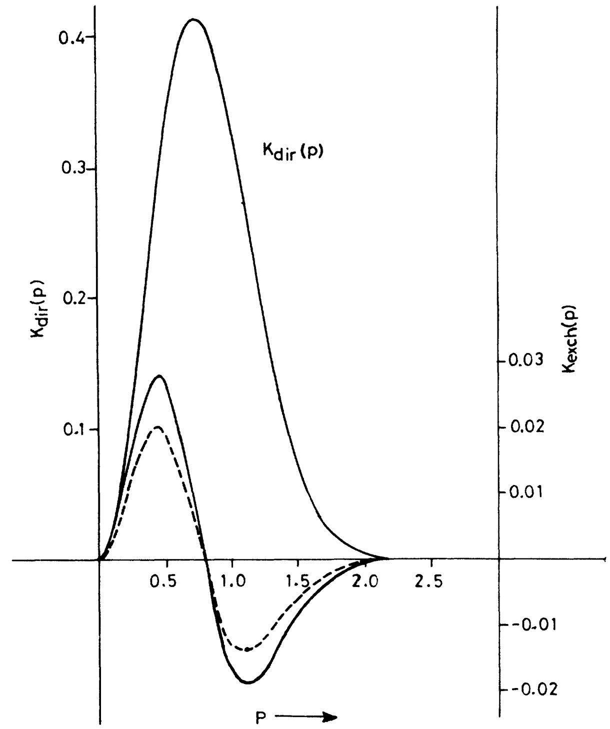

FIG. 7. Quark momentum distribution in a static nucleon, together with density dependent corrections in a Fermi gas ($K_F=1.4$ fm^{-1}) without correlations (solid line), as well as with realistic (Ref. 6) hardcore correlation parameters $\gamma^2=1.1$ fm^{-2} and $\beta^2=0.68$ fm^{-2} (dashed line). The nucleon rms radius $b=0.8$ fm.

C. Structure function

The object which is directly measured in lepton-nucleus deep inelastic scattering is $F_2(x,Q^2)$ and not $K(p)$. In the absence of a QCD solution to the nucleus one needs a model to relate the two. The requisite connection was made in Ref. 2,

$$R(x)=\frac{F_2^T(x)}{F_2^{T^*}(x)} =\frac{\int_{p_{\min}}^{\infty}dp\,p[K_{\text{dir}}(p)+K_{\text{exch}}(p)]}{\int_{p_{\min}}^{\infty}dp\,pK_{\text{dir}}(p)}\ . \tag{4.12a}$$

Here T is the nuclear target, while T^* is another hypothetical target with identical quantum numbers, but in which the nucleons are sufficiently removed from each other. The lower limit of integration is

$$P_{\min}=\frac{(xM+\varepsilon_0)^2-m^2}{2(xM+\varepsilon_0)}\ . \tag{4.12b}$$

The expressions for $K_{\rm dir}$ and $K_{\rm exch}$ obtained in the preceding section are so simple that one can immediately perform the integration above to get

$$R(x)=1-\left[\frac{3}{\pi}\right]^{1/2}\frac{\alpha^3}{6}U\left[1-\left[\frac{32}{63}\right]^{1/2}e^{-(3/14)b^2p_{\rm min}^2}-\left[\frac{2}{9}\right]^{1/2}e^{-(3/2)b^2p_{\rm min}^2}\right]. \tag{4.13}$$

This is an important result of this paper. Since $p_{\rm min}$ increases monotonically with x, $R(x)$ deviates from unity progressively. As discussed in Ref. 2, the model has validity only in a restricted range, $0.3 \lesssim x \lesssim 0.8$.

V. RESULTS AND DISCUSSION

Plotted in Fig. 8 are the results for the EMC ratio $R(x)$ in nuclear matter with $K_F=1.4$ fm^{-1}, and with hardcore correlations included. In the absence of quark exchange this ratio would simply be unity. Also plotted for reference is the range of experimental data for ^{56}Fe. Clearly the present calculation does not exactly explain the data—and if it had then that would be quite fortuitous. Iron is not nuclear matter and surface effects are nonnegligible; Fermi motion does make a difference; and use of Gaussian wave functions for the nucleon is only qualitatively correct. Nevertheless, there is no doubt that the sign, as well as the rough magnitude of the EMC effect, is correctly predicted here. This upholds the conclusion arrived at in Ref. 2: A lot of the EMC effect could merely be a consequence arising from the indistinguishability of quarks belonging to different nucleons.

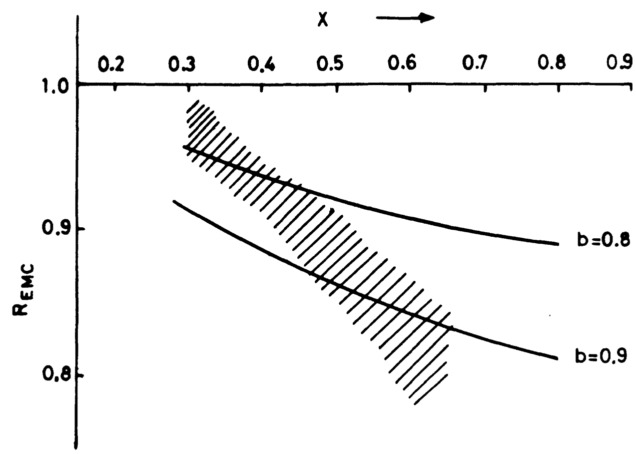

FIG. 8. The EMC ratio $R(x)$ predicted in the quark exchange model [Eq. (4.13)] for a correlated infinite Fermi gas. For comparison, a range of data on ^{56}Fe is also shown.

It is also hoped that the formalism developed in this paper will enable calculations of various other effects coming from quark exchange in heavy nuclei. Hitherto such calculations have been limited to $N=2,3,4$ only.[7]

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