Quark fragmentation functions in a diquark model for proton and Λ hyperon production

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A simple quark-diquark model for nucleon and Λ structure is used to calculate leading twist light-cone fragmentation functions for a quark to inclusively decay into p or Λ. The parameters of the model are determined by fitting to the known deep-inelastic structure functions of the nucleon. When evolved from the initial to the final Q^2 scale, the calculated fragmentation functions are in remarkable agreement (for $z > 0.4$) with those extracted from partially inclusive ep and e^+e^- experiments at high energies. Predictions are made, using no additional parameters, for longitudinally and transversely polarized quarks to fragment into p or Λ.

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Fragmentation of partons produced in a high-energy process into definite hadronic states is a problem in QCD of considerable current interest. Parton fragmentation, in a sense, is dual to the process of uncovering hadron structure using high momentum probes. This duality has recently been emphasized by Jaffe and Ji [1,2]. These authors have made a detailed exploration of the spin, chirality, and twist structures of fragmentation using the fact that parton fragmentation functions can be expressed in QCD as matrix elements of quark and gluon field operators at light-cone separations. Indeed, a one-to-one correspondence between fragmentation functions and parton distribution functions can be established. While this is a useful step towards understanding the nature of the hadronization process, it certainly does not solve the problem—the calculation of distribution functions belongs to the realm of nonperturbative QCD and therefore can only be modeled. Fragmentation functions are even harder to model theoretically, and existing models using strings and shower algorithms, etc. [3] are complicated and involve many parameters.

In this paper we explore an alternate, and much simpler, description for the conversion of a fast quark into specific hadrons. The starting point is an expression bilinear in quark field operators which defines the unpolarized and polarized fragmentation functions in terms of the light-cone momentum fraction $z = P^+/k^+$ (see Fig. 1):

$$4\frac{\hat{f}_1(z)}{z}p^+ = \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \mathrm{Tr}\langle 0|\gamma^+\psi(0)|PX\rangle\langle PX|\bar{\psi}(\lambda n)|0\rangle, \tag{1}$$

$$4\frac{\hat{g}_1(z)}{z}p^+ S\cdot n = \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \mathrm{Tr}\langle 0|\gamma^+\gamma_5\psi(0)|PS_{\parallel}X\rangle\langle PS_{\parallel}X|\bar{\psi}(\lambda n)|0\rangle, \tag{2}$$

and

$$4\frac{\hat{h}_1(z)}{z}S_\perp p^+ = \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \mathrm{Tr}\langle 0|\gamma^+\gamma^\perp\gamma_5\psi(0)|PS_\perp X\rangle\langle PS_\perp X|\bar{\psi}(\lambda n)|0\rangle. \tag{3}$$

In the above, p^μ and n^μ are null vectors with $p^2 = n^2 = p^- = n^+ = 0$ and $p\cdot n = 1$. In terms of these, the four-momentum of the produced hadron, whose rest frame we shall take as our reference frame, is $P^\mu = p^\mu + \frac{1}{2}M^2 n^\mu$. The spin vector of the produced spin-$\frac{1}{2}$ hadron is $S^\mu = S\cdot np^\mu + S\cdot pn^\mu + MS_\perp^\mu$, and $\gamma^+ = (1/\sqrt{2})(\gamma^0+\gamma^3)$. The gauge $n\cdot A = A^+ = 0$ is used. A summation over X is implicit and covers all possible states which can be populated by quark fragmentation. The quark operators

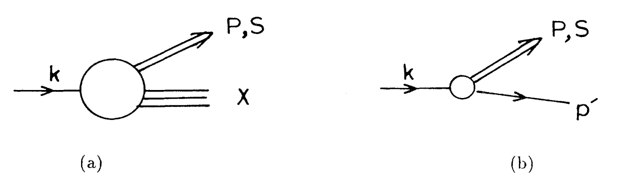

FIG. 1. The fragmentation of a quark with momentum k (a) into a specific hadron $(p, \Lambda, \ldots)$ is modeled (b) with a quark-diquark-hadron vertex.

are at equal $x_\perp$ and x^+ but are separated by a variable light-cone distance $x^- = \lambda n^-$. To interpret $\hat{f}_1(z)$ physically, define a composite operator $C^\dagger(P)$ which creates a hadron of a specific type and momentum from the vacuum, $|P\rangle = C^\dagger(P)|0\rangle$. Using completeness of X, it is easy to show that

$$\hat{f}_1(z) = \int [dk]\delta\left(\frac{k^+}{P^+} - \frac{1}{z}\right)\frac{\langle k|C^\dagger(P)C(P)|k\rangle}{\langle k|k\rangle}, \quad (4)$$

where

$$[dk] = \frac{d^2k_T dk^+}{(2\pi)^3 2k^+}. \quad (5)$$

This shows that $\hat{f}_1(z)$ is the probability of finding a given hadron with fixed z in a quark, irrespective of the transverse quark momentum.

The matrix elements in Eqs. (1) and (2) need to be modeled as they cannot be calculated *ab initio*. Perhaps the simplest assumption is that the quark fragments into a baryon and antidiquark [Fig. 1(b)]. This may be reasonable provided that z is not too far from 1, i.e., the produced hadron carries away most of the momentum of the quark. The amplitude for the process is

$$\langle PS|\bar{\psi}|0\rangle = \bar{u}(PS)\Phi\frac{i}{\not{k} - m}, \quad (6)$$

where Φ is the vertex that connects the quark with the two outgoing particles. Φ is a matrix in Dirac space, and its most general form is rather complicated since it involves several unknown form factors. Instead, we shall be guided by the investigations of Meyer and Mulders [4], and Melnitchouk, Schreiber, and Thomas [5], who have calculated nucleon structure functions in the diquark model and obtained rather good fits to deep inelastic scattering at all data except small x. Following Ref. [4], we take

$$\Phi = \phi(P^2, p'^2, k^2)\mathcal{I}, \quad (7)$$

where $\mathcal{I}$ is the unit matrix and only scalar diquarks are included in the vertex. Inserting Eqs. (6) and (7) into Eqs. (1)–(3) and neglecting the quark mass m in the numerator yields

$$\hat{f}_1(z) = \frac{1}{4(1-z)}\int_0^\infty \frac{dk_\perp^2}{(2\pi)^2}\frac{M^2 + z^2k_\perp^2}{(k^2 - m^2)^2}|\phi|^2, \quad (8)$$

$$\hat{g}_1(z) = \frac{1}{8(1-z)}\int_0^\infty \frac{dk_\perp^2}{(2\pi)^2}\frac{M^2 - z^2k_\perp^2}{(k^2 - m^2)^2}|\phi|^2, \quad (9)$$

and

$$\hat{h}_1(z) = \frac{1}{8(1-z)}\int_0^\infty \frac{dk_\perp^2}{(2\pi)^2}\frac{M^2}{(k^2 - m^2)}^2|\phi|^2 . \quad (10)$$

The fragmenting quark, which is highly off shell, has its k^2 given by

$$k^2 = \frac{M^2}{z} + \frac{m_d^2 + zk_\perp^2}{1-z}. \quad (11)$$

The integrals are logarithmically divergent and need to be damped by a suitable form factor. Following Ref. [5] we choose ϕ to be

$$\phi(k^2) = N\frac{k^2 - m^2}{(k^2 - \Lambda^2)^2}. \quad (12)$$

Here Λ is a mass parameter whose choice will be decided upon later. N is a normalization constant which is fixed by the requirement that the quark state be normalized covariantly, $\langle k|k'\rangle = (2\pi)^3 2k^0\delta^3(\mathbf{k} - \mathbf{k}')$.

Let us now concentrate on the spin-isospin structure of the produced baryon. In the notation of Meyer and Mulders [4], the SU(6) wave function for a spin-up proton is

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}\overset{\uparrow}{u}S_0^0 + \frac{1}{\sqrt{18}}\overset{\uparrow}{u}T_0^0 - \frac{1}{3}\overset{\downarrow}{u}T_0^1 - \frac{1}{3}\overset{\uparrow}{d}T_1^0 - \sqrt{\frac{2}{9}}\overset{\downarrow}{d}T_1^1. \quad (13)$$

S and T are quark combinations representing scalar and vector diquark, respectively, e.g., $T_1^1 = \overset{\uparrow}{u}\overset{\uparrow}{u}$, etc. There is, of course, no explicit coupling to vector diquarks in the vertex equation (7) although, at the expense of some complication, it could be put in. We follow instead the easy route suggested by the calculations of Ref. [4] wherein it was assumed that the $S(x)$ and $T(x)$ distribution functions are of identical form but differ because the S and T diquarks have somewhat different masses m_S and m_V, as well as mass parameters Λ_S and Λ_V. The $N - \Delta$ mass difference leads to $m_V - m_S \approx 200$ MeV. Following a similar logic, define $\hat{S}(z)$ and $\hat{T}(z)$ to be fragmentation functions where the emitted antidiquark is S and T, respectively:

$$\begin{aligned}\hat{S}(z) &= \hat{f}_1(z) \text{ with } m_d = m_S, \Lambda = \Lambda_S,\\ \hat{T}(z) &= \hat{f}_1(z) \text{ with } m_d = m_V, \Lambda = \Lambda_V.\end{aligned} \quad (14)$$

The polarized quantities $\Delta\hat{S}(z)$ and $\Delta\hat{T}(z)$ are defined similarly with $\hat{f}_1(z)$ replaced by $\hat{g}_1(z)$ in the above. $\Delta_1\hat{S}(z)$ and $\Delta_1\hat{T}(z)$ are defined with $\hat{f}_1(z)$ replaced by $\hat{h}_1(z)$. The u and d ragmentation functions are readily seen to be

$$\begin{aligned}\hat{f}_{1u}^P(z) &= \tfrac{1}{2}\hat{S}(z) + \tfrac{1}{6}\hat{T}(z),\\ \hat{f}_{1d}^P(z) &= \tfrac{1}{3}\hat{T}(z),\\ \hat{g}_{1u}^P(z) &= \tfrac{1}{2}\Delta\hat{S}(z) - \tfrac{1}{18}\Delta\hat{T}(z),\\ \hat{g}_{1d}^P(z) &= -\tfrac{1}{9}\Delta\hat{T}(z),\\ \hat{h}_{1u}^P(z) &= \tfrac{1}{2}\Delta_1\hat{S}(z) - \tfrac{1}{18}\Delta_1\hat{T}(z) ,\\ \hat{h}_{1d}^P(z) &= -\tfrac{1}{9}\Delta_1\hat{T}(z).\end{aligned} \quad (15)$$

The same approach can be followed for the production of a Λ hyperon from u, d, or s quarks. The hyperon wave function is

$$|\Lambda\uparrow\rangle = \frac{1}{\sqrt{12}}\Bigg(\overset{\uparrow}{u}\,\mathcal{T}^0 - \overset{\uparrow}{d}\,\mathcal{T}^0 - \sqrt{2}\,\overset{\downarrow}{u}\,\mathcal{T}^1 + \sqrt{2}\,\overset{\downarrow}{d}\,\mathcal{T}^1 + \overset{\uparrow}{u}\,\mathcal{S}^0 + \overset{\uparrow}{d}\,\mathcal{S}^0 - 2\,\overset{\uparrow}{s}\,S^0\Bigg). \tag{16}$$

Here $\mathcal{S}$ and $\mathcal{T}$ are diquarks with one s and either a u or d quark, while S is a ud system as before. Define new fragmentation functions

$$\hat{\mathcal{S}}(z) = \hat{f}_1(z) \text{ with } m_d = m_{\mathcal{S}}, \Lambda = \Lambda_S,$$
$$\hat{\mathcal{T}}(z) = \hat{f}_1(z) \text{ with } m_d = m_{\mathcal{T}}, \Lambda = \Lambda_V,$$
$$\hat{S}(z) = \hat{f}_1(z) \text{ with } m_d = m_S, \Lambda = \Lambda_S. \tag{17}$$

The quantities $\Delta\hat{\mathcal{S}}(z), \Delta\hat{\mathcal{T}}(z)$, and $\Delta\hat{S}(z)$ are defined similarly with $\hat{f}_1(z)$ replaced by $\hat{g}_1(z)$, and $\Delta_1\hat{\mathcal{S}}(z)$, $\Delta_1\hat{\mathcal{T}}(z)$, $\Delta_1\hat{S}(z)$ are defined with $\hat{f}_1(z)$ replaced by $\hat{h}_1(z)$. In terms of these the fragmentation of u, d, and s into Λ is given by

$$\hat{f}^{\Lambda}_{1u}(z) = \hat{f}^{\Lambda}_{1d}(z) = \tfrac{1}{4}\hat{\mathcal{T}}(z) + \tfrac{1}{12}\hat{\mathcal{S}}(z),$$
$$\hat{f}^{\Lambda}_{1s}(z) = \tfrac{1}{3}\hat{S}(z), \tag{18}$$

and, for the polarized quantities,

$$\hat{g}^{\Lambda}_{1u}(z) = \hat{g}^{\Lambda}_{1d}(z) = \tfrac{1}{12}[\Delta\hat{\mathcal{S}}(z) - \Delta\hat{\mathcal{T}}(z)],$$
$$\hat{g}^{\Lambda}_{1s}(z) = \tfrac{1}{3}\Delta\hat{S}(z),$$
$$\hat{h}^{\Lambda}_{1u}(z) = \hat{h}^{\Lambda}_{1d}(z) = \tfrac{1}{12}[\Delta_1\hat{\mathcal{S}}(z) - \Delta_1\hat{\mathcal{T}}(z)],$$
$$\hat{h}^{\Lambda}_{1s}(z) = \tfrac{1}{3}\Delta_1\hat{S}(z). \tag{19}$$

We now turn to a discussion of numerical results and comparisons with fragmentation data, where available. The diquark model for fragmentation of u, d quarks to protons needs, as input, the scalar and vector diquark masses m_S, m_V as well as the cutoffs Λ_S, Λ_V. Consistent with the conclusions of Melnitchouk *et al.* [5], we find that a reasonable fit to nucleon structure functions can be achieved with the choice $m_S = 900$ MeV, $m_V = 1100$ MeV, $\Lambda_S = 840$ MeV, and $\Lambda_V = 925$ MeV. This fixes all the parameters needed for the model, and one may use it for calculating fragmentation rates into the protons. Figure 2 shows $\hat{f}^P_{1u}(z)$ and $\hat{f}^P_{1d}(z)$ calculated using Eqs. (8) and (12). Since these are scale-dependent quantities, one must specify the scale as well. It appears reasonable to take the initial scale to be $m_d + m_p$, where m_p is the proton mass. This, of course, is the minimum off-shell mass of the fragmenting quark. Evolution of the fragmentation function to the experimental scale can be performed exactly as for quark distribution function. For example,

$$\mu\frac{\partial}{\partial\mu}f^P_i(z,\mu) = \sum_j \int_z^1 \frac{dy}{y} P_{i\to j}\left(\frac{z}{y},\mu\right) f^P_j(y,\mu), \tag{20}$$

where $P_{i\to j}(x,\mu)$ is the Altarelli-Parisi function for the splitting of the parton of type i into a parton of type j with longitudinal momentum fraction x. In principle, the sum in Eq. (20) extends over gluons and antiquarks as well. At large z this is hopefully small and so we ignore this, including only $P_{u\to u}(x,\mu)$ and $P_{d\to d}(x,\mu)$, where

$$P_{u\to u}(x,\mu) = \frac{2\alpha_s(\mu)}{3\pi}\left(\frac{1+x^2}{1-x}\right)_+ . \tag{21}$$

The + function is defined in the usual way:

$$f(x)_+ = f(x) - \delta(1-x)\int_0^1 dx' f(x'). \tag{22}$$

The evolved fragmentation function $\hat{f}^P_u(z,Q^2)$ is compared in Fig. 3 with the European Muon Collaboration (EMC) data [6] extracted from μ-p and μ-D scattering. The agreement is fairly good even down to rather small values of z, where model has no reason to be valid. Without changing parameters, predictions for $\hat{g}_1$ and $\hat{h}_1$ are shown in Figs. 4 and 5, respectively.

The calculation of u, d, s quark fragmentation into Λ hyperons proceeds identically with the sole change of increasing the diquark mass by $M_\Lambda - M_p = 176$ MeV if the outgoing diquark contains a strange quark. The SU(6) wave functions then determine the relative magnitudes of $\hat{f}_1$, $\hat{g}_1$, and $\hat{h}_1$, which are plotted, respectively, in Figs. 6–8. That the produced Λ carries the spin of the fragmenting s quark is apparent from Fig. 7. The transverse fragmentation function $\hat{h}_1$ for u, d, $s \to \Lambda$, if mea-

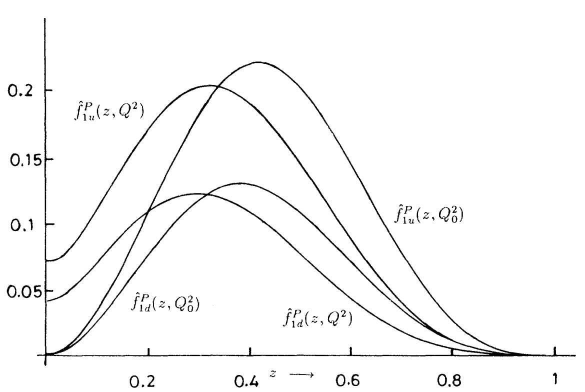

FIG. 2. Fragmentation functions for u and d quarks to go into a proton, calculated in the diquark model, evolved from the initial scale $Q_0^2 = (m_d + m_p)^2$ to $Q^2 = 80$ GeV^2.

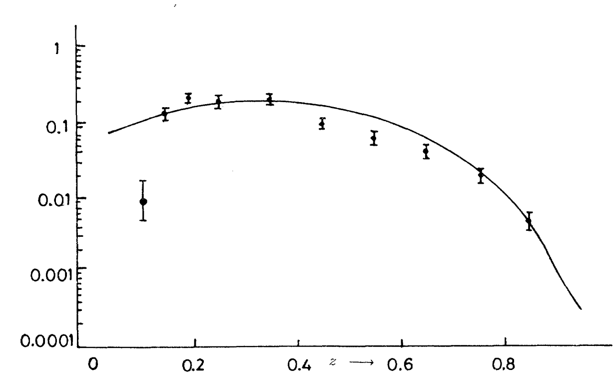

FIG. 3. Evolved diquark model fragmentation function $f^P_u(z,Q^2)$ compared against EMC [6] data extracted from μ-p and μ-D scattering.

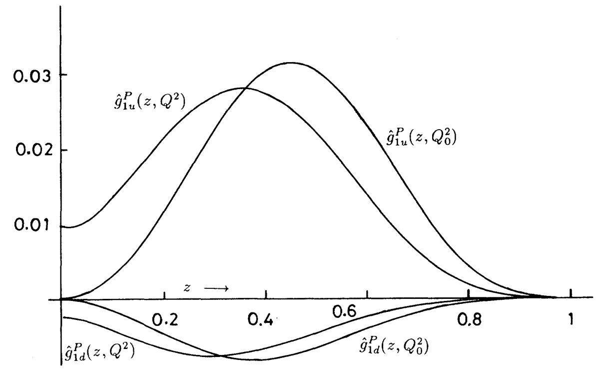

FIG. 4. As in Fig. 2, except that the fragmentation function $\hat{g}_1$ is plotted here.

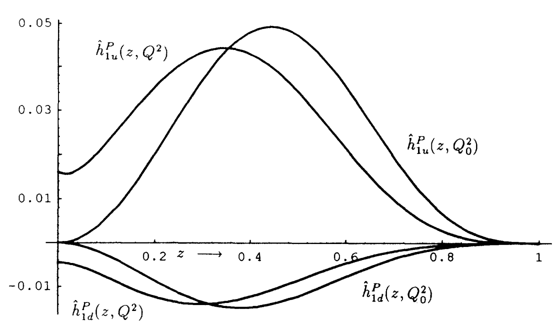

FIG. 5. As in Fig. 2, except that the fragmentation function $\hat{h}_1$ is plotted here.

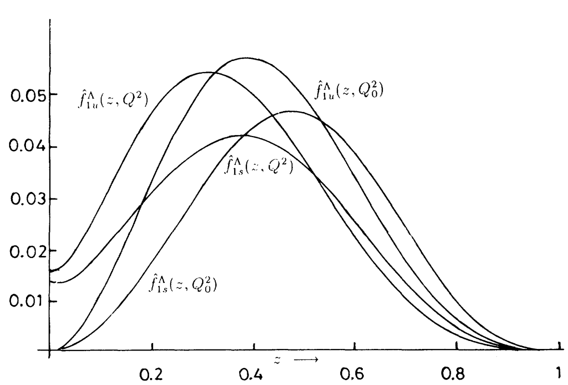

FIG. 6. Diquark model calculation for u, d, s quarks to fragment into Λ hyperons. The initial scale $Q_0^2 = (m_d + m_\Lambda)^2$ and final scale $Q^2 = 80\ \mathrm{GeV}^2$ fragmentation functions are shown.

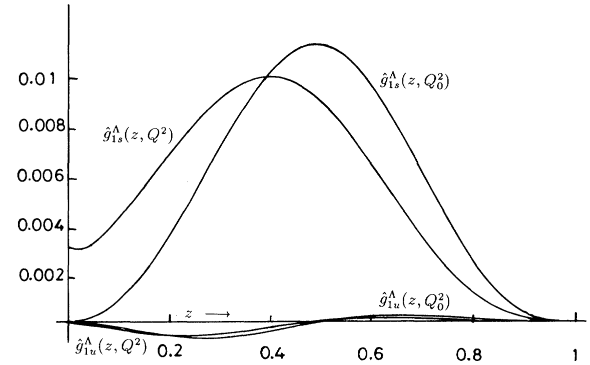

FIG. 7. As in Fig. 6, except that the fragmentation function $\hat{g}_1$ is plotted here.

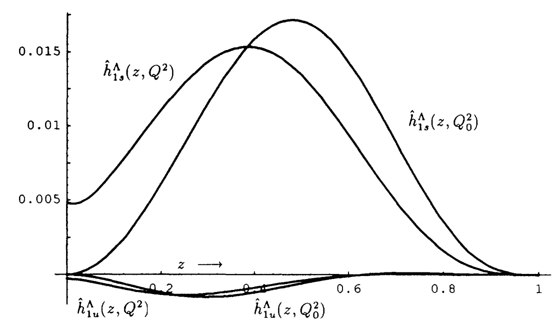

FIG. 8. As in Fig. 6, except that the fragmentation function $\hat{h}_1$ is plotted here.

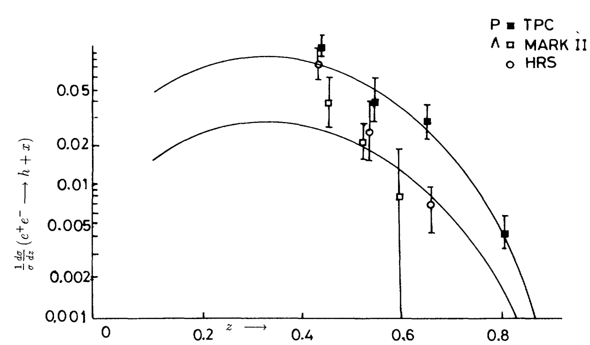

FIG. 9. Production of protons and hyperons in e^+e^- collisions at c.m. energy of about 30 GeV. The solid curves are predictions of the diquark fragmentation model, and the data is from TPC [8], HRS [9], and Mark II [10] collaborations.

sured, would be a good tool for uncovering the transverse distribution function $h_1(x)$ [1,7] of the proton. While no experimental spin data exists, data from e^+e^- collisions at large c.m. energies can be compared with predictions of the present model provided we assume that only u, d quarks lead to proton production, and only u, d, s quarks lead to Λ production. Figure 9 contains a comparison of theory to experiment. Again the evolution has been performed to a final scale of $Q^2 = 80$ GeV^2.

To conclude, we have investigated a simple fragmentation model for a quark to go into a p, Λ and the appropriate antidiquark using a vertex with a suitable form-factor determined by fitting to deep inelastic data. The good agreement with experiment, even down to rather small z values where this agreement is surely fortuitous, suggests that one is perhaps using the right effective degrees of freedom for p, Λ production from quarks. At the same time, it is obvious that one is far from a complete description of fragmentation; the model gives zero chance for antiproton production from a quark. Nevertheless, it has good predictive power, and we have calculated the spin-fragmentation functions $\hat{g}_1$ and $\hat{h}_1$ for p, Λ production. It will be interesting to see how well these are eventually borne out by experiment.

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[1] R. L. Jaffe and X. Ji, Phys. Rev. Lett. **71**, 2547 (1993).
[2] X. Ji, Phys. Rev. D **49**, 114 (1994).
[3] See, for example, V. D. Barger and R. J. N. Phillips, *Collider Physics* (Addison-Wesley, Reading, MA, 1987).
[4] H. Meyer and P. J. Mulders, Nucl. Phys. **A528**, 589 (1991).
[5] W. Melnitchouk, A. W. Schreiber, and A. W. Thomas, Phys. Rev. D **49**, 1183, 1994.
[6] M. Arneodo *et al.*, Nucl. Phys. **B321**, 541 (1989).
[7] X. Artru and M. Mekhfi, Z. Phys. C **45**, 669 (1990).
[8] TPC, H. Aihara *et al.*, Phys. Rev. Lett. **53**, 2378 (1984).
[9] HRS Collaboration, M. Derrick *et al.*, Phys. Rev. D **35**, 2639 (1987).
[10] Mark II Collaboration, C. de la Vaissiere *et al.*, Phys. Rev. Lett. **54**, 2071 (1985).