Quark orbital-angular-momentum distribution in the nucleon

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We introduce gauge-invariant quark and gluon angular momentum distributions after making a generalization of the angular momentum density operators. From the quark angular momentum distribution, we define the gauge-invariant and leading-twist quark orbital angular momentum distribution \( L_q(x) \). The latter can be extracted from data on the polarized and unpolarized quark distributions and the off-forward distribution \( E(x) \) in the forward limit. We comment upon the evolution equations obeyed by this as well as other orbital distributions considered in the literature. [S0556-2821(99)02501-1]

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It has long been suspected that the quark orbital angular momentum (OAM) plays an important role in the structure of the nucleon, even though empirical evidence suggests that the constituent quarks are predominantly in the s-wave state in the naive quark model. For instance, Sehgal noticed, following the work of Ellis and Jaffe, that the quark spin falls short in accounting for the spin of the proton. He considered seriously the possibility of a sizable quark OAM filling in this gap. Ratcliffe studied the angular momentum conservation in the parton splitting processes, pointing out that the OAM can be generated by parton helicity during the scale evolution. Since the European Muon Collaboration (EMC) data on polarized deep-inelastic scattering, a large number of theoretical papers have appeared on the spin structure of the proton, some of which have directly and indirectly considered the issue of the quark OAM structure of the proton, even though empirical evidence suggests that the constituent quarks are predominantly in the s-wave state in the naive quark model. For instance, Sehgal and in leading-order QCD perturbation theory. Based on this, it was concluded that the fraction of the nucleon spin carried by quarks is about \( \frac{1}{2} \) in the asymptotically large scale limit. Taking into account the polarized deep inelastic scattering (DIS) data, it was observed that approximately \( \frac{1}{6} \) of the nucleon spin resides in the quark OAM in the asymptotic limit. In Ref. [6], a proposal was made to gain access experimentally to the quark OAM at a finite scale \( \mu^2 \) by studying virtual Compton scattering in the deep-inelastic limit.

Recently, Hägler and Schäfer and Harindranath and Kundu studied the quark OAM distribution as a function of Feynman \( x \) in the nucleon. They considered a tower of operators,

\[
\bar{\psi} \gamma^+(x^1 i \gamma^2 - x^2 i \gamma^1) i D^+ \cdots i D^+ \psi,
\]

whose matrix elements are used to define the moments of the quark OAM distribution. They derived the one-loop evolution equation which has a homogeneous term the same as the evolution kernel for the unpolarized quark distribution [11], generalizing a result found earlier for the first moment [6,8]. The mixing of the quark OAM distribution with the quark helicity distribution can also be related to the polarized and unpolarized Altarelli-Parisi kernels, as elucidated recently by Teryaev.

In this paper, we introduce an alternative quark OAM distribution. Our distribution is motivated from the quark angular momentum distribution defined from the form factors or matrix elements of spin-independent twist-2 operators. The new quark OAM distribution can be extracted from the polarized and unpolarized quark distributions and the off-forward distributions \( E(x) \) introduced in Ref. [8]. One important feature of the new distribution is that it is leading twist and its evolution is completely determined by that of polarized and unpolarized quark and gluon distributions to all orders in perturbation theory.

We start with the familiar spin-independent, twist-2, flavor-singlet (summing over quark flavors) quark operators,

\[
O_{q}^{\beta \mu_1 \cdots \mu_n}(\xi) = \frac{1}{i} \bar{\psi} \gamma^\beta \hat{D}^{\mu_1} \cdots \hat{D}^{\mu_n} \psi(\xi),
\]

where all indices \( \beta, \mu_1, \ldots, \mu_n \) are symmetric and all traces have been subtracted, as indicated by the parentheses \( (\cdots) \). Note that the \( n = 1 \) operator is just the traceless part of the energy-momentum tensor of quarks. From \( O_{q}^{\beta \mu_1 \cdots \mu_n} \), we define a new tower of twist-2 operators which generalize the angular momentum density,

\[
M_{q}^{\alpha \beta \mu_1 \cdots \mu_n}(\xi) = \xi^\alpha O_{q}^{\beta \mu_1 \cdots \mu_n}(\xi) - \xi^\beta O_{q}^{\alpha \mu_1 \cdots \mu_n}(\xi) - (\text{traces}),
\]

where the subtraction of traces ensures that \( M_{q}^{\alpha \beta \mu_1 \cdots \mu_n} \) belongs to an irreducible representation of the Lorentz group. The matrix element of the above operator in the nucleon state \( |PS\rangle \), where \( P \) and \( S \) are respectively the momentum and polarization vectors \( (P^2 = M^2, S^2 = -M^2, S \cdot P = 0) \), is

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where \([\alpha\beta]\) indicates antisymmetrization of the two indices. Here terms with derivatives on the \(\delta\) function have been omitted because they depend on the center-of-mass motion of the nucleon. The matrix element \(J_{qn}\) generalizes the nucleon spin carried by quarks \(J_{q1}\). In fact, we can define from it a quark angular momentum distribution \(J_q(x)\) such that

\[
J_q(x) = \int_{-1}^{1} J_q(x) \, dx = J_{qn}.
\]  

\(J_q(x)\) at negative \(x\) represents the antiquark contribution. If we limit the support to \(0 < x < 1\), the angular momentum distribution is \(J_q(x) + J_q(-x)\). Obviously one can extend all of the above discussions to gluons and define the gluon angular momentum distribution \(J_g(x)\).

The matrix element \(J_{qn}\) or, equivalently, the angular momentum distribution \(J_q(x)\) can be obtained from the off-forward distribution \(H(x, \xi, t)\) and \(E(x, \xi, t)\) in the forward limit \(\xi = t = 0\). According to Ref. [8], these distributions are defined in terms of the off-forward matrix element of the light-cone string operator,

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P' \left| \bar{\phi} \left( -\frac{\lambda}{2}\right) \right| \mathcal{P} \exp \left( -ig \int \frac{d\alpha}{\lambda^2} A(\alpha n) \right) \bar{\phi} \left( \frac{\lambda}{2} \right) \right| P \right\rangle = H_q(x, \xi, t) \bar{U}(P') \mathcal{U}(U(P)) + E_q(x, \xi, t) \bar{U}(P') \mathcal{U}(U(P)) \frac{i g^{\mu\nu} \eta_{\mu\nu}}{2M} U(P),
\]

where \(\bar{U}(P')\) and \(U(P)\) are the Dirac spinors, \(\Delta = P' - P, \bar{P} = (P' + P)/2\), and \(t = \Delta^2\). The vector \(n\) along the light-cone \((n^2 = 0)\) is conjugate to \(\bar{P}\) in the sense that \(\bar{P} \cdot n = 1\). Taking the \(n\)th moments of the above expression, we get

\[
n_{\mu_1 \mu_2 \ldots \mu_n} (P') \langle O_{\mu_1 \mu_2 \ldots \mu_n} | P \rangle = H_{qn}(x, \xi, t) \bar{U}(P') \mathcal{U}(U(P)) + E_{qn}(x, \xi, t) \bar{U}(P') \mathcal{U}(U(P)) \frac{i g^{\mu\nu} \eta_{\mu\nu}}{2M} U(P),
\]

where

\[
H_{qn}(x, \xi, t) = \int_{-1}^{1} dx x^{n-1} H_q(x, \xi, t) dx, \quad E_{qn}(x, \xi, t) = \int_{-1}^{1} dx x^{n-1} E_q(x, \xi, t) dx
\]

are the moments of the off-forward distributions. To find their relation with \(J_{qn}\), we write down all possible elastic form factors of the twist-2 operators after using the constraints from Lorentz symmetry and parity and time reversal invariance:

\[
\langle P' | O_{\mu_1 \ldots \mu_n} | P \rangle = \bar{U}(P') \gamma^{\mu_1} \mathcal{U}(U(P)) \sum_{i = 0}^{[n-1]/2} A_{qn,2i}(t) \Delta_{\mu_2 \ldots \mu_{2i+1}} \bar{P}^{\mu_{2i+2}} \ldots \bar{P}^{\mu_n})
\]

\[
+ \bar{U}(P’) \frac{\alpha^{\mu_1 \ldots \mu_n}}{2M} U(P) \sum_{i = 0}^{[n-1]/2} B_{qn,2i}(t) \Delta_{\mu_2 \ldots \mu_{2i+1}} \bar{P}^{\mu_{2i+2}} \ldots \bar{P}^{\mu_n})
\]

\[
+ C_{qn}(t) \text{Mod}(n + 1, 2) \frac{1}{M} \bar{U}(P') U(P) \Delta^{(\mu_1 \ldots \mu_n)}.
\]

For \(n \geq 1\), even or odd, there are \(n + 1\) form factors. \(C_{qn}(t)\) is present only when \(n\) is even. Substituting the above expression into Eq. (4), we have

\[
J_{qn} = \frac{1}{2} [A_{qn+1,0}(0) + B_{qn+1,0}(0)].
\]

Contracting both sides of Eq. (9) with \(n^{\mu_1} \ldots n^{\mu_n}\) and comparing the result with Eq. (7), we obtain the desired relation

\[
J_{qn} = \frac{1}{2} [H_{qn+1}(0,0) + E_{qn+1}(0,0)].
\]

Clearly \(H_q(x, 0, 0)\) is just the singlet quark distribution \(q(x)\). For convenience we abbreviate \(E_q(x, 0, 0)\) as \(E_q(x)\). Then the above relation can be translated to one for the quark angular momentum distribution,
\[ J_q(x) = \frac{1}{x}[q(x) + E_q(x)]. \]  \hspace{1cm} (12)

The distribution \( q(x) \) has been extracted from high-energy scattering with good accuracy \([13]\). However, \( E(x) \) is unknown at present except for its first moment (see below).

Since forming spatial moments of an operator as in Eq. (4) does not change its short distance behavior, the evolution equation for the angular momentum distributions \( J_q(x) \) and \( J_g(x) \) is exactly the same as that for the unpolarized quark and gluon distributions,

\[ \frac{d}{d \ln \mu^2} \left( \frac{J_{nq}(\mu)}{J_{nq}(\mu)} \right) = \left( \frac{\gamma_{qq}(n+1)}{\gamma_{gq}(n+1)} \frac{\gamma_{gq}(n+1)}{\gamma_{gg}(n+1)} \right) \left( \frac{J_{nq}(\mu)}{J_{nq}(\mu)} \right) \]  \hspace{1cm} (13)

where the anomalous dimension matrix is a perturbation series in \( \alpha_s \), and the leading order result can be found in Ref. \([11]\). Of course, according to Eq. (12) the same conclusion follows from the evolution of the off-forward parton distributions \([8]\).

Now we can define the quark orbital angular momentum distribution by looking at the structure of \( M_q^{\alpha\beta\mu_1\cdots\mu_n} \). Since it is already an irreducible operator under Lorentz transformations, the generalized OAM operator that it contains is not irreducible and hence the nucleon matrix element of the latter cannot be a Lorentz scalar. This is an intrinsic limitation to the significance of the notion of the orbital angular momentum in a relativistic quantum theory. However, if we are limited to a class of coordinates in which the nucleon has a definite helicity, we still can define the orbital angular momentum and its distribution in Feynman \( x \) \([14]\). In the following discussion, we assume the nucleon is moving in the \( z \) direction and is polarized in the helicity \( +1/2 \) state. We consider only the specific components \( \alpha = 1, \beta = 2, \mu_1 = \cdots = \mu_n = + \) of the \( M_q \) operator and take its spatial integral,

\[ \int d^3 \xi M_{12+\cdots+n}^{q(2+\cdots+n)} = \int d^3 \xi \left[ \bar{\psi} \gamma_i D^+ \cdots D^+ \psi + \bar{\psi} \gamma^+ i D^2 \cdots i D^+ \psi + \cdots + \bar{\psi} \gamma^+ i D^n \cdots i D^+ \psi \right] - (1 \leftrightarrow 2). \]  \hspace{1cm} (15)

The first term in the brackets can be further manipulated,

\[ \bar{\psi} \gamma_i D^+ \cdots D^+ \psi = \frac{1}{n} \left[ \bar{\psi} (\gamma^2 i D^+ - \gamma^+ i D^2) i D^+ \cdots D^+ \psi + \cdots + \bar{\psi} i D^+ \cdots i D^+ (\gamma^2 i D^+ - \gamma^+ i D^2) \psi \right] \]

\[ + \frac{1}{n} \left[ \bar{\psi} \gamma^+ i D^2 \cdots i D^+ \psi + \cdots + \bar{\psi} \gamma^+ i D^n \cdots i D^+ \psi \right]. \]  \hspace{1cm} (16)

The second term in brackets on the right hand side is the same as the remaining terms in Eq. (15). So we concentrate on the first term in the above equation. Using the identity

\[ \gamma^2 i D^+ - \gamma^+ i D^2 = \frac{1}{2} [i D^+ \gamma^2 - \gamma^+ \gamma^2 i D^+], \]  \hspace{1cm} (17)

and partially integrating the first derivative term, we have

\[ \int d^3 \xi \bar{\psi} (\gamma^2 i D^+ - \gamma^+ i D^2) i D^+ \cdots D^+ \psi + \cdots + \bar{\psi} i D^+ \cdots i D^+ (\gamma^2 i D^+ - \gamma^+ i D^2) \psi \]

\[ = - i \frac{n}{2} \int d^3 \xi \bar{\psi} \gamma^+ \gamma \gamma^2 i D^+ \cdots D^+ \psi + \int d^3 \xi [ \bar{\psi} \gamma^+(x^1 \gamma^2)(i F^\mu_\nu) \gamma_\mu i D^+ \cdots i D^+ \psi ] \]

\[ + \cdots + \int d^3 \xi [ \bar{\psi} \gamma^+(x^1 \gamma^2)(i F^\mu_\nu) \gamma_\mu \gamma^0 \psi ] \]

\[ = \int d^3 \xi [ \bar{\psi} \gamma^+(x^1 \gamma^2)(i F^\mu_\nu) \gamma_\mu \gamma^0 \psi ] \]  \hspace{1cm} (18)

where we have used the quark equations of motion to eliminate some terms.

Putting all the pieces together, we have

\[ \int d^3 \xi M_{12+\cdots+n}^{q(2+\cdots+n)} = S_{q+\cdots+n}^+ + L_{q+\cdots+n}^+ + \Delta L_{q+\cdots+n}^+, \]  \hspace{1cm} (19)
The matrix element of $S^{++...+}$ is clearly related to the polarized quark distribution $\Delta q(x)$ which has been the major focus of the polarized deep-inelastic scattering experiments in the last ten years [4,7]. Here $L^{++...+}$ seems to be a natural gauge-invariant generalization of the OAM operator. However, from its definition, it is clear that $L^{++...+}$ contains not only the leading twist but also the higher-twist contributions. In particular, under a change of renormalization scale, it mixes with $\Delta L^{++...+}$. Only the sum $L^{++...+} + \Delta L^{++...+}$ evolves as a leading-twist operator. Therefore, we define the leading-twist, gauge-invariant generalization of the OAM operator as

$$L^{++...+} = \bar L^{++...+} + \Delta L^{++...+}. \tag{21}$$

Its matrix element in the nucleon state is

$$\langle PS|L^{++...+}|PS\rangle = \frac{2L_{qn}}{n+1} 2P^+ \cdots P^+ (2\pi)^3 \delta^3(0). \tag{22}$$

Let us emphasize again that the simple structure of the matrix element is only possible in the specific class of frames that we defined earlier. From the above, the OAM distribution function $L_q(x)$ can be defined as

$$\int_{-1}^{1} L_q(x)x^{n-1}dx = L_{qn}, \tag{23}$$

which can be expressed in terms of the quark angular momentum and helicity distributions,

$$L_q(x) = J_q(x) - \frac{1}{2} \Delta q(x). \tag{24}$$

The definition of the matrix elements of $S^{++...+}$ and its relation to the quark helicity distribution is standard,

$$\langle PS|S^{++...+}|PS\rangle = \frac{\Sigma_n}{n+1} 2P^+ \cdots P^+ (2\pi)^3 \delta^3(0), \tag{25}$$

$$\int_{-1}^{1} dx x^{n-1} \Delta q(x) = \Sigma_n. \tag{25}$$

Combining Eqs. (12),(24), we have

$$L_q(x) = \frac{1}{2} \{ x[q(x)+E_q(x)] - \Delta q(x) \}. \tag{26}$$

Thus, the quark OAM distribution can be determined entirely from the polarized and unpolarized singlet quark distribution and the off-forward distribution $E(x)$.

The evolution equation for $L_{qn}(\mu)$ is straightforward. The evolution of $\Sigma_n$ is well known:

$$\frac{d}{d\ln \mu} \Sigma_n = \Delta \gamma_{qq}(n) \Sigma_n + \Delta \gamma_{qg}(n) \Delta g_n, \tag{27}$$

where $\Delta g_n$ are the moments of the gluon helicity distribution and the anomalous dimensions can be found in Ref. [11]. Combining the above with Eq. (13), we have the evolution equation for $L_{qn}$,

$$\frac{d}{d\ln \mu} L_{qn} = \gamma_{qq}(n+1)L_{qn} + \frac{1}{2} \left[ \gamma_{qq}(n+1) - \Delta \gamma_{qq}(n) \right] \Sigma_n$$

$$+ \gamma_{qg}(n+1)J_{g\perp} - \frac{1}{2} \Delta \gamma_{qg}(n) \Delta g_n, \tag{28}$$

which is again valid to all orders in perturbation theory.

In view of the above discussion, we now have a new perspective on the evolution equations obtained by Hagler and Schäfer, and Harindranath and Kundu. The quark OAM operators defined in these works (we will call it $L^{++...+}$) differ from our $L^{++...+}$ by terms which depend on transverse polarizations of the gluon potentials ($A^1,A^2$). Our calculation indicates that these extra terms do not mix in the $L^{++...+}$ operator at the one-loop level, although at higher-loops they must do so that $L^{++...+}$ maintain an all-order simple evolution in Eq. (28). Therefore at the one-loop level, $L_{qn}^{++...+}$ and $J_{g\perp}$ have an identical homogeneous part in their evolution equation. Taking into account the extra factor of $2/(n+1)$ in our definition of the OAM matrix elements, we deduce that $L_{qn}^{++...+} + \Delta \Sigma_n/(n+1)$ has the same homogeneous evolution as $J_{g\perp}$ at one-loop level. A similar analysis for the gluon angular momentum shows that $F_{g\perp}^{0} + 2\Delta g_{n}/(n+1)$ has the same homogeneous evolution as $J_{g\perp}$ at one loop. From these, we immediately deduce that

$$S^{++...+} = \frac{2}{n+1} \int d^3 \bar q \gamma^\alpha \frac{\Sigma^3}{2} \bar i D^+ \cdots i D^+ \psi,$$

$$\bar L^{++...+} = \frac{1}{n} \int d^3 \bar q \gamma^\alpha \left( x^1 i D^2 - x^2 i D^1 \right) i D^+ \cdots i D^+ \psi + \cdots + \bar \psi \gamma^\alpha i D^+ \cdots i D^+ \left( x^1 i D^2 - x^2 i D^1 \right) \psi,$$

$$\Delta L^{++...+} = \frac{1}{n(n+1)} \int d^3 \bar q \gamma^\alpha \left( x^1 \gamma^2 - x^2 \gamma^1 \right) \left( igF^\alpha \gamma_\rho \right) i D^+ \cdots i D^+ \psi + \cdots + \bar \psi \gamma^\alpha i D^+ \cdots i D^+ \left( x^1 \gamma^2 - x^2 \gamma^1 \right) \left( igF^\alpha \gamma_\rho \right) \psi]. \tag{20}$$
indeed small, L q also provide access to the distribution. Virtual Compton scattering has been proposed as a way to measure it is not guaranteed to have a similar simple structure at higher loops. Because of these operators, the evolution equation at one loop. The explicit terms agree with the known result [9, 10]. The ellipsis denotes the matrix elements of operators with explicit transverse gluon fields which are present even at the one-loop level. The explicit terms agree with the known result [9, 10]. Because of these operators, the evolution equation is not guaranteed to have a similar simple structure at higher loops.

To determine the quark OAM distribution, one has to measure $E(x)$ from off-forward hard scattering processes. Deeply virtual Compton scattering has been proposed as a way to measure it [8]. High-energy diffractive vector-meson production can also provide access to the distribution [15]. The first moment of $E(x)$ is related to the anomalous magnetic moment:

$$\int_{-1}^{1} E(x) dx = \kappa_u + \kappa_d + \kappa_s .$$

The experimental data on this are $0.33 \pm 0.39$, a fairly small number [16]. This could indicate a small size of $E(x)$. If $E(x)$ is indeed small, $L_q(x)$ can be determined entirely in terms of $\Delta q(x)$ and $q(x)$. However, there is no sound theoretical reason to neglect $E(x)$ at present.

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