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Spin structure functions of nuclei in the QCD parton recombination model

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The multipole $L=2$ structure function, measurable in deeply inelastic scattering of unpolarized leptons off a polarized $J\geq 1$ nuclear target, is a good indicator of exotic quark-gluon components in the nucleus. We present here estimates of this structure function for two different classes of nuclei—light nuclei describable in an independent-particle model approach and heavy nuclei described by slowly rotating collective variables. An estimate of the exotic effects is made within the context of a model wherein gluons and quarks from separate nucleons fuse together to alter the parton densities relative to that in isolated nucleons.

I. INTRODUCTION

Deep-inelastic scattering of leptons from polarized nuclear targets with spin $J\geq 1$ has recently been investigated theoretically[1,2] and is likely to be the subject of experimental investigations in the near future.[3] An interesting feature of $J\geq 1$ targets is the existence of higher multipole structure functions whose measurement may provide important clues and insight into the quark-gluon structure of nuclei which go well beyond that provided by measurements of unpolarized nuclear structure functions. The $L=$even multipole structure functions[1,2] ${}^J_L F_1(x,Q^2)$, obtained from scattering unpolarized leptons from a nucleus of spin J, are also expected to show nuclear effects similar to the European Muon Collaboration effect.[4] In this paper, working within a parton recombination model based on QCD, we shall study various nuclear spin-dependent structure functions, specifically ${}^J_L F(x,Q^2)$ with $L=2$ for $J=\frac{3}{2}$ and $\frac{7}{2}$ target nuclei. For $J=\frac{3}{2}$ we shall consider a lithium nucleus consisting of a spinless core coupled to a single valence nucleon. The $J=\frac{7}{2}$ target nucleus is holmium-165. It is a highly deformed nucleus with its body-fixed-symmetry axis precessing around the total angular momentum **J**, which, in turn, precesses around the laboratory frame z axis.

The modifications of ${}^J_2 F_1(x,Q^2)$ that will be studied here are those that arise from the fusion of partons originating from different nucleons. Following standard arguments,[5,6] consider the nuclear parton distribution in the infinite-momentum frame (IMF). In the IMF the nucleus is Lorentz contracted in the $\hat{\mathbf{z}}$ direction and has a longitudinal size $\Delta z_A \approx 2R/\gamma \approx 2mR/p$, where m and p are the nucleon mass and momentum, respectively. On the other hand, the longitudinal size of a sea parton (whether quark or gluon) is $\Delta z \approx 1/xp$, where x is the fraction of the nucleon's momentum carried by the parton. For small x, Δz exceeds the size of the nucleus, and shadowing occurs for $x\leq 1/2mR$; i.e., the nuclear parton distribution is no longer A times the nucleon parton distribution, $q^A \neq Aq^N(x,Q^2)$. For sufficiently small x the sea quarks and gluons from a given nucleon extend along the entire length of the nucleus in the $\hat{\mathbf{z}}$ direction. Using this physical picture, Mueller and Qiu[5,6] derived a modified Altarelli-Parisi equation valid for small x as well, which successfully predicted the slow falloff with Q^2 of the unpolarized structure function. In contrast, vector-dominance models give a $1/Q^2$ decrease, which is incompatible with the data. Mueller and Qiu did not, however, attempt to make a prediction of the modification as a function of x at a fixed value of Q^2. This task was attempted by Close, Qiu, and Roberts.[7] Their model, which we shall use as the basis of our calculations, is briefly reviewed below.

A. Parton recombination model

Close, Qiu, and Roberts[7] have developed a parton recombination model which generates shadowing of structure functions. In this model a parton from one nucleon leaks into a neighboring nucleon and fuses with partons of the latter. Prior to fusion the partons have undergone normal QCD evolution [Fig. 1(a)]. The change in nuclear parton-number density due to the parton recombination process [Fig. 1(b)] is given by

$$\Delta P(x,Q^2)=\int dx_1 dx_2 T^{(2)}(x_1,x_2,l_T^2) \times \frac{g^2}{l_T^2}\Gamma_{p_1p_2\to p_3}(x_1,x_2,x_1+x_2) \times \delta(x-x_1-x_2)\ . \tag{1.1}$$

Using the old-fashioned perturbation theory in the IMF, the parton-parton fusion function is calculated to be

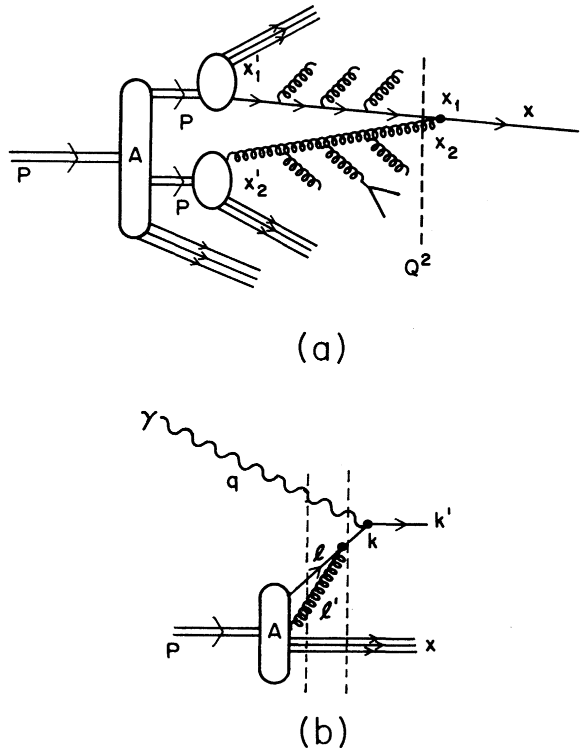

FIG. 1. (a) Fusion of two partons from neighboring nucleons after undergoing normal QCD evolution, prior to recombination. (b) The leading parton recombination diagram used to define the two-parton-number density and to calculate the fusion function $\Gamma_{gq\to q}$.

$$\frac{g^2}{l_T^2}\Gamma_{p_1p_2\to p_3}=\frac{E_k}{E_l+E_{l'}}|M_{ll'\to k}|^2 \times\left[\frac{1}{E_l+E_{l'}-E_k}\right]^2\left[\frac{1}{2E_k}\right]^2 , \tag{1.2a}$$

where the parton momenta are

$$l=\left[x_1p+\frac{l_T^2}{2x_1p},l_T,x_1p\right] ,$$
$$l'=\left[x_2p+\frac{l_T^2}{2x_2p},-l_T,x_2p\right] , \tag{1.2b}$$
$$k=(x_3p,0_T,x_3p) .$$

p is the average momentum per nucleon in the IMF, l_T is the transverse momentum of the fusing partons, and the momentum fraction carried by the final parton is $x_3=x_1+x_2$. The transverse size of these partons is $\Delta b_\perp\sim 1/|Q|$. Therefore, l_T^2 is of order Q^2. One might expect parton fusion to disappear as $1/Q^2$, but the modified evolution equations of Mueller and Qiu[5] lead to a much slower falloff. Three basic subprocesses give rise to changes in the nuclear parton distributions: quark-gluon fusion, quark-antiquark fusion, and gluon-gluon fusion (Fig. 2). The fusion function for each process can

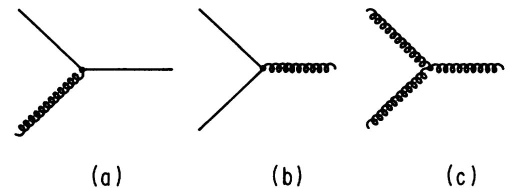

FIG. 2. Fusion diagrams contributing to $\Delta P(x,Q^2)$. (a) Quark-gluon fusion. (b) Quark-antiquark fusion. (c) Gluon-gluon fusion.

be derived from basic QCD vertices using old-fashioned perturbation theory. Finally, $T^{(2)}(x_1,x_2,l_T^2)$ in Eq. (1.1) is the two-parton number density and can be approximated by

$$T^{(2)}(x_1,x_2,Q^2)=\mathcal{C}P_1(x_1,Q^2)P_2(x_2,Q^2) . \tag{1.3}$$

$P_1(x_1,Q^2)$ and $P_2(x_2,Q^2)$ are the number densities of the two fusing partons. The factor $\mathcal{C}$ controls the magnitude of shadowing since it determines the number of shadowed nucleons inside a nucleus. In terms of the configuration shown in Fig. 3,

$$\mathcal{C}\approx\int d^3R\, d^3r\, \rho(\mathbf{R})\rho(\mathbf{r})\delta^2(\mathbf{B}-\mathbf{b}) . \tag{1.4}$$

$\rho(\mathbf{r})$ is the nucleon-number density and the δ function restricts the impact parameters of nucleons in volume elements d^3R and d^3r to be the same.

Temporarily ignoring sea quarks and antiquarks (these are reinstated in our full calculations), the modification to the nuclear quark density due to the process (a) in Fig. 2 is

$$\Delta q(x)=2\frac{g^2}{Q^2}\mathcal{C}\int dx_1dx_2q(x_1)G(x_2) \times\Gamma_{qg\to q}(x_1,x_2,x_1+x_2) \times[\delta(x-x_1-x_2)-\delta(x-x_1)] , \tag{1.5}$$

where the quark-gluon-fusion function is given by

$$\Gamma_{qg\to q}(x_1,x_2,x_1+x_2) =\frac{1}{6}\frac{x_1x_2}{(x_1+x_2)^2}\left[\frac{1+[x_1/(x_1+x_2)]^2}{1-[x_1/(x_1+x_2)]}\right] . \tag{1.6}$$

In terms of this fusion function, the modification to $\sum_f e_f^2q_f(x)$ is

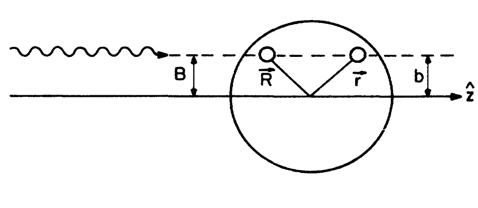

FIG. 3. Picture of shadowing used to evaluate $\mathcal{C}$, the number of shadowed nucleons.

$$\sum_f e_f^2 \Delta q_f(x) = \frac{1}{6x} K \int_0^x \frac{dx_2}{x_2} x_2 G(x_2) W(x_2) \left\{ F_2(x-x_2)\left[1+\left[\frac{x-x_2}{x}\right]^2\right] - F_2(x)\left[\frac{x}{x+x_2}\right]\left[1+\left[\frac{x}{x+x_2}\right]^2\right]\right\}$$
$$- \frac{1}{6x} K F_2(x)\int_x^1 \frac{dx_2}{x_2} x_2 G(x_2)W(x_2)\left[\frac{x}{x+x_2}\right]\left[1+\left[\frac{x}{x+x_2}\right]^2\right], \qquad (1.7)$$

where $K=\mathcal{C}g^2/Q_0^2$ is the shadowing parameter at a fixed value Q_0^2. $W(x)$ is a cutoff function which controls the extent to which gluons from one nucleon leak into another. It is taken to be[7]

$$W(x)=\exp(-\tfrac{1}{2}m^2\langle z^2\rangle x^2)\ .$$

$\langle z^2\rangle^{1/2}$ is the leakage parameter and is varied from zero to 2.4 fm. $\langle z^2\rangle^{1/2}=0$ corresponds to no cutoff—gluons and sea quarks are allowed to extend along the entire length of the nucleus.

B. Multipole structure functions

Deep-inelastic scattering from polarized nuclear targets can measure exotic effects in nuclei.[1,2] In the Bjorken limit all information about the spin-J target nucleus resides in $(2J+1)$-parton distribution functions $q_S^{JH}(x)$, where $q_S^{JH}(x)$ is the probability of finding a quark with momentum fraction x and spin component s along the z axis in a hadron moving with infinite momentum along the z axis, with spin H along the z axis. (The z axis is along the photon momentum $\mathbf{q}$.)

Multipole structure functions are defined in terms of these parton distributions as

$$\sum_{H=-J}^{J} (-1)^{J-H} C^{J\ J\ L}_{H\,-H\,0}\, q_\uparrow^{JH}(x) = \begin{cases} {}^J_L F_1(x), & L \text{ even},\ 0\le L\le 2J\ , \\ {}^J_L g_1(x), & L \text{ odd},\ 1\le L\le 2J\ . \end{cases} \qquad (1.8)$$

We are interested in studying the modification of the symmetric spin-dependent function ${}^J_2F_1(x)$ due to parton fusion, where

$${}^J_2F_1(x)=\sum_{H=-J}^{J} (-1)^{J-H} C^{J\ J\ 2}_{H\,-H\,0}\, q_\uparrow^{JH}(x)\ . \qquad (1.9)$$

${}^J_2F_1(x)$ is independent of quark spin since it weights $q_\uparrow^{JH}$ and $q_\downarrow^{JH}$ equally. However, it is a measure of the difference in quark distribution due to different spin orientations of the target nucleus. In general, one has no means by which to calculate the quark distributions $q_\uparrow^{JH}$; this would require solution of the bound-state QCD problem for a nucleus. Consequently, one must work within the context of specific models to compute this.

II. ${}^J_2F_1(x)$ FOR A LIGHT NUCLEUS (^{7}Li)

As a simplified model for the ^{7}Li target, we shall assume a single nucleon in the $1p_{3/2}$ level outside a spherical closed core. This ignores core polarization effects (which could be included with some effort), but it is quite adequate for a first calculation. The single nucleon thus carries all the spin of the nucleus. Our convolution model calculation includes relativity by keeping the lower components of the nucleon spinor. However, for the shadowing correction to the convolution result, we need to use relatively cruder information about the nuclear wave function—it is only a matter of computing geometrical overlaps and using the parton recombination model of Close, Qiu, and Roberts.[7]

A. Convolution model

The convolution approximation states that $q_\uparrow^{JH}(x)$ can be expressed in terms of nucleon parton distributions (Fig. 4) according to

$$q_\uparrow^{JH}(x)=\int_0^{M_A/m} dy \int_0^1 dz\, \delta(x-yz) \times \sum_{s=\uparrow,\downarrow} f_s^{JH}(y)\, q_\uparrow^{1/2s}(z)\ . \qquad (2.1)$$

$f_\uparrow^{JH}(y)$ is the probability of finding a nucleon with momentum fraction y and spin up (along the z axis) in a target nucleus with spin H along the z axis. M_A is the mass of the target nucleus and m is the nucleon mass. $q_\uparrow^{1/2s}(z)$ is the nucleon parton distribution for a spin-up quark in a spin-s ($s=\uparrow,\downarrow$) nucleon. The multipole structure function ${}^J_LF_1(x)$ defined in (1.8) is

$${}^J_LF_1(x)=\int_0^{M_A/m} dy \int_0^1 dz\, \delta(x-yz) F_1(z)\, {}^J_Lf(y)\ , \qquad (2.2)$$

where

$${}^J_Lf(y)=\sum_{H=-J}^{J} (-1)^{J-H} C^{J\ J\ L}_{H\,-H\,0}\, f^{JH}(y)\ , \qquad (2.3)$$

with $f^{JH}(y)=f_\uparrow^{JH}(y)+f_\downarrow^{JH}(y)$ and $F_1(z)=\frac{1}{2}[q_\uparrow^{1/2\uparrow}(z)$

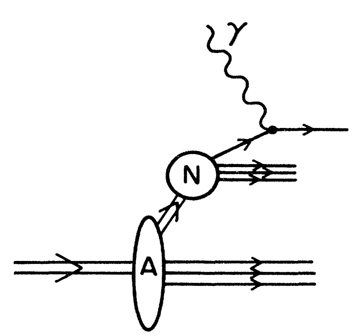

FIG. 4. Convolution model for parton distributions.

$+q_{\downarrow}^{1/2\uparrow}(z)]$ is the nucleon's spin-averaged structure function.

We shall now apply the above formulas to calculate ${}^{3/2}_{2}F_1(x)$ for a ^{7}Li target. The smearing function ${}^{3/2}_{2}f(y)$, which has to be convoluted with the nucleon's structure function $F_1(z)$ to obtain ${}^{3/2}_{2}F_1(x)$, is[1,2]

$$ {}^{3/2}_{2}f(y) = -\frac{1}{2\pi}\int d^3p\,|\phi(p)|^2\left[1+\frac{p\cos\theta}{m}+\frac{p^2}{4m^2}\right] \times \mathcal{P}_2(\cos\theta)\delta\left[y-\frac{p\cos\theta+E}{m}\right] . \quad (2.4) $$

For a semirelativistic nucleon with $p^2/m^2 \ll 1$, ${}^{3/2}_{2}f(y)$ can be expanded in a distribution about $y=1$. Including terms through order $O(p^2/m^2)$, ${}^{3/2}_{2}f(y)$ is

$$ {}^{3/2}_{2}f(y) = -\frac{2}{15}\left\langle\frac{p^2}{m^2}\right\rangle[-2\delta'(y-1)+\delta''(y-1)] + O\left[\frac{p^4}{m^4}\right] , \quad (2.5a) $$

where

$$ \left\langle\frac{p^2}{m^2}\right\rangle = \int_0^\infty dp\,\frac{p^4}{m^2}|\phi(p)|^2 . \quad (2.5b) $$

Using Eqs. (2.2) and (2.5a), it is straightforward to obtain ${}^{3/2}_{2}F_1(x)$:

$$ {}^{3/2}_{2}F_1(x) = -\frac{2}{15}\left\langle\frac{p^2}{m^2}\right\rangle[2xF_1'(x)+x^2F_1''(x)] . \quad (2.6) $$

A numerical evaluation of Eq. (2.6) is shown in Fig. 5. The quark distributions entering $F_1(x)$ were chosen to be from Duke and Owens[8] (set 1 at Q^2 = 4 GEV^2) and $\langle p^2/m^2\rangle = 0.04$.

B. Fusion corrections to ${}^{3/2}_{2}F_1(x)$

In the absence of any correlation between core nucleons and the valence proton, ${}^{3/2}_{2}F_1(x)$ would get no

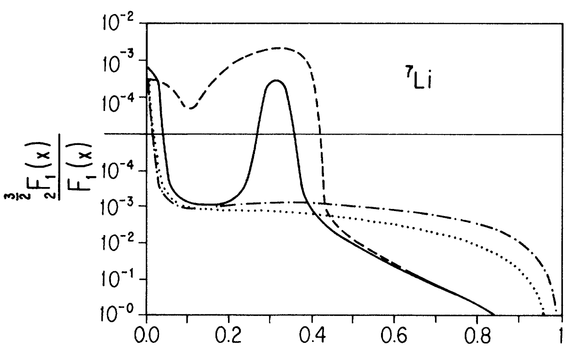

FIG. 5. (i) ${}^{3/2}_{2}F_1(x)/F_1(x)$ in the convolution model for ^{7}Li (dashed curve). (ii) Parton fusion correction (dotted curve) with no cutoff, i.e., $\langle z^2\rangle^{1/2}=0$—partons extend over the entire length of the nucleus. (iii) Parton fusion correction (dot-dashed curve) with $\langle z^2\rangle^{1/2}=2.4$ fm. (iv) Sum of convolution plus fusion correction with $\langle z^2\rangle^{1/2}=0$ (solid curve).

contribution from the spinless core, and one would obtain only the value predicted in Sec. II A. However, the shadowing of the valence nucleon by the core alters the situation. The extent by which the core affects the nuclear-spin parton distribution is determined by the dimensionless quantity K:

$$ K = \frac{g^2}{Q^2}\int d^3R\, d^3r\, \rho_{\text{core}}(\mathbf{R})\rho_N(\mathbf{r})\delta^2(\mathbf{B}-\mathbf{b}) . \quad (2.7) $$

We may take the core density $\rho_{\text{core}}(\mathbf{r})$ to be a simple Gaussian:

$$ \rho_{\text{core}} = \frac{A_0}{(\pi R_0^2)^{3/2}}e^{-r^2/R_0^2} , \quad (2.8) $$

where A_0 is the total number of core nucleons and R_0 the core radius. For the valence nucleon density ρ_N, we consider a harmonic-oscillator wave function corresponding to the $p_{3/2}$ state, viz.,

$$ \psi_{3/2\,1\,m}(r) = N\left[\frac{r}{r_0}\right]e^{-(1/2)(r/r_0)^2}\mathcal{Y}_{3/2\,1\,m}(\hat{\mathbf{r}}) ; \quad (2.9) $$

where N is a normalization constant, r_0 is the radius of the nucleon orbital, and $\mathcal{Y}_{jlm}(\hat{\mathbf{r}})$ is a vector spherical harmonic. This gives the valence nucleon density for $m=\frac{3}{2}$ and $\frac{1}{2}$:

$$ \rho_N^{m=3/2} = |\psi_{3/2\,1\,3/2}|^2 = \frac{r^2e^{-(r/r_0)^2}}{r_0^5\pi^{3/2}}\sin^2\theta , \quad (2.10a) $$

$$ \rho_N^{m=1/2} = |\psi_{3/2\,1\,1/2}|^2 = \frac{r^2e^{-(r/r_0)^2}}{3r_0^5\pi^{3/2}}(\sin^2\theta+4\cos^2\theta) . \quad (2.10b) $$

The shadowing parameter K^{JH} for a spin-J nucleus with spin projection H is, therefore,

$$ K^{3/2\;3/2} = \frac{g^2A_0}{Q_0^2\pi}\frac{R_0^2}{(r_0^2+R_0^2)^2} , \quad (2.11a) $$

$$ K^{3/2\;1/2} = \frac{g^2A_0}{Q_0^2\pi}\left[\frac{R_0^2}{3(r_0^2+R_0^2)^2}+\frac{2}{3(r_0^2+R_0^2)}\right] . \quad (2.11b) $$

For a numerical estimate of the shadowing parameters, we use a starting value of $Q_0^2=4$ GeV^2 and $\alpha(Q_0^2)=g^2/4\pi=0.327$. Typically, $r_0=1.19(A^{1/3}-0.44)^{1/2}$ and $R_0^2=1.83r_0^2$.

The modification due to shadowing of q^{JH}, the spin-averaged parton distribution, is given by Eq. (1.7) with the entire spin dependence residing in the shadowing parameter K^{JH}. Hence

$$ \sum_f e_f^2\Delta q_f^{JH} = K^{JH}\sum_f e_f^2\Delta q_f . \quad (2.12) $$

From Eq. (1.9), ${}^{3/2}_{2}F_1(x)$ is expressed in terms of quark distributions

$$ {}^{3/2}_{2}F_1(x) = \tfrac{1}{2}(q_\uparrow^{3/2\;3/2}-q_\uparrow^{3/2\;1/2}-q_\uparrow^{3/2\;-1/2}+q_\uparrow^{3/2\;-3/2}) = \tfrac{1}{2}(q^{3/2\;3/2}-q^{3/2\;1/2}) , \quad (2.13) $$

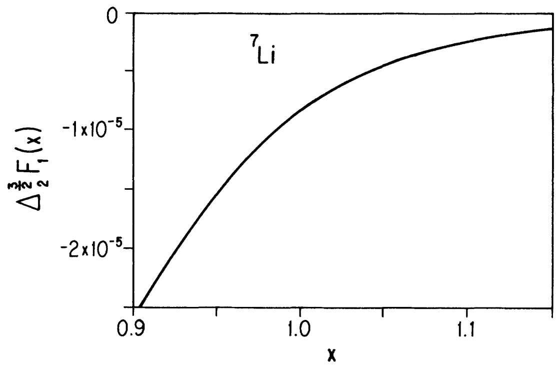

FIG. 6. $\Delta\,{}_2^{3/2}F_1(x)$ for ^{7}Li and x beyond 1.

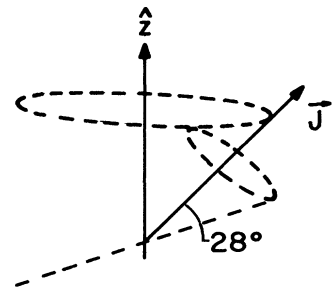

FIG. 7. ^{165}Ho with its body-fixed axis precessing about **J**, which in turn precesses about the external magnetic-field direction.

where $q^{JH}=q_{\uparrow}^{JH}+q_{\downarrow}^{JH}$ and the sum over flavor has been suppressed. The shadowing correction to ${}_2^{3/2}F_1(x)$ follows from Eqs. (2.11), (2.12), and (2.13):

$$\Delta\,{}_2^{3/2}F_1=\tfrac{1}{2}\sum_f e_f^2(\Delta q_f^{3/2\;3/2}-\Delta q_f^{3/2\;1/2})$$
$$=\tfrac{1}{2}(K^{3/2\;3/2}-K^{3/2\;1/2})\sum_f e_f^2\Delta q_f\ . \qquad (2.14)$$

C. Results

The convolution-model prediction and shadowing corrections to ${}_2^{3/2}F_1(x)$ are depicted in Fig. 5. In each case comparison is made with the nucleon structure function $F_1(x)$. Expectedly, the convolution-model prediction is rather small—we can see from Eq. (2.6) that the size of this is governed by the small parameter $\langle p^2/m^2\rangle\approx 0.04$. Moreover, there is effectively only one nucleon which is polarized. Parton recombination effects are comparable to the convolution value for $x<0.4$ and are relatively insensitive to the cutoff function $W(x)$, especially in the region of interest. For $x<0.2$ the modification to ${}_2^{3/2}F_1(x)$ is considerably larger than the convolution-model prediction. Hence this effect is possibly discernible in a measurement of ${}_2^{3/2}F_1(x)$. Also, the absorption of a low-x parton by a $x\approx 1$ parton causes the structure function to extend beyond $x=1$ (Fig. 6).

III. ${}_2^{J}F_1(x)$ FOR A HEAVY NUCLEUS (^{165}Ho)

Many heavy nuclei are highly deformed and spinning, and hence orientable. As a typical heavy nucleus, consider ^{165}Ho, which has a large intrinsic quadrupole moment of about 7.6×10^{-24} cm^2 and a large deformation parameter $\beta\simeq 0.3$. Unfortunately, the uncertainty principle prevents use of the entire deformation in asymmetry experiments even if the nuclei are 100% polarized. If **J** is the nuclear-spin vector ($J=\frac{7}{2}$), the intrinsic (or body-fixed) axis precesses around **J** (see Fig. 7) at an angle of about 28°. Further, **J** precesses around the external magnetic-orienting field at an angle of 28°. Hence the intrinsic axis cannot be aligned sharply with the external field. We shall now proceed to calculate convolution and shadowing contributions to the $L=2$ structure function of ^{165}Ho.

A. Convolution model

In order to obtain the smearing function ${}_2^{7/2}f(y)$, the momentum distribution of the nucleons in the laboratory frame is required. For ^{7}Li this was straightforwardly done because one had a simple wave function in the laboratory frame with good angular momentum. But for heavy deformed nuclei, one has only the body-fixed (intrinsic) nuclear wave function Φ for which the value of the angular momentum is highly indeterminate, i.e., $\langle\Phi J^2\Phi\rangle\gg 1$. This implies that Φ is a superposition of many angular momentum eigenstates. However, we require nuclear wave functions of definite spin. In order to project out angular momentum eigenstates ψ_J from Φ, we use Villar's formalism[9] and define the projection operator

$$P_{KH}^{J}=\int d\Omega\, D_{KH}^{\dagger J}(\Omega)\mathcal{R}(\Omega)\ . \qquad (3.1)$$

$\mathcal{R}(\Omega)$ is a rotation operator, and $D_{KH}^{J}(\Omega)$ the associated rotation matrix. Operating P_{KH}^{J} on a many-particle determinantal wave function $\Phi(x_1,x_2,\ldots,x_A)$ yields a function ψ_{KH}^{J}:

$$\psi_{KH}^{J}(x_1,\ldots,x_A)=P_{KH}^{J}\Phi(x_1,\ldots,x_A)\ , \qquad (3.2a)$$

which has the properties

$$J^2\psi_{KH}^{J}=J(J+1)\psi_{KH}^{J}\ , \qquad (3.2b)$$

$$J_z\psi_{KH}^{J}=H\psi_{KH}^{J}\ . \qquad (3.2c)$$

H is, as before, the projection of **J** along the laboratory $\hat{\mathbf{z}}$ axis, while K is the projection along the body-fixed 3 axis. Our objective now is to use this formalism to calculate the laboratory-frame momentum distribution of nucleons given a many-body wave function in the body-fixed frame. This is carried out explicitly in the Appendix. The result of the calculation is

$$\langle\rho(\mathbf{p})\rangle_{JH}=\sum_l C_{H0H}^{JlJ}C_{K0K}^{JlJ}f_l(p)Y_{l0}(\hat{\mathbf{p}})\ , \qquad (3.3)$$

where

$$f_l(p)=\sum\int d\hat{\mathbf{p}}'\,\phi^*(p,\hat{\mathbf{p}}')Y_{l0}(\hat{\mathbf{p}}')\phi(p,\hat{\mathbf{p}}')\ . \qquad (3.4)$$

The summation in (3.4) is over all the nucleons. The single-particle wave function is expressed in terms if Nilsson orbitals:

$$\phi_{N\Omega}=\sum_{L\Lambda} a_{L\Lambda}|NL\Lambda\Sigma\rangle \ ,$$

where N specifies the oscillator shell and $\Omega=\Lambda+\Sigma$ is the projection of $\mathbf{J}$ along the body-fixed 3-axis. In momentum space,

$$\phi_{N\Omega}(p,\hat{\mathbf{p}}')=\sum_{L\Lambda} a_{L\Lambda}\phi_N(p)Y_{L\Lambda}(\hat{\mathbf{p}}')\chi_\Sigma \ ,$$

and, therefore,

$$f_l(p)=\sum \left[\frac{2l+1}{4\pi}\right]^{1/2} \phi_N^2(p) \sum_{LL'\Lambda} a_{NL'\Lambda}a_{NL\Lambda}C^{LlL'}_{\Lambda 0\Lambda}C^{LlL}_{000} \ . \tag{3.5}$$

The smearing function ${}^{7/2}_{\ \ 2}f(y)$ is calculated from the above to be

$${}^{7/2}_{\ \ 2}f(y)=\int d^3p\, {}^{7/2}_{\ \ 2}\rho(\mathbf{p})\delta\left[y-\frac{p\cos\theta+E}{m}\right] , \tag{3.6a}$$

where

$${}^{7/2}_{\ \ 2}\rho(\mathbf{p})=\sum_{H=-7/2}^{7/2}(-1)^{7/2-H}C^{7/2\ \ 7/2\ \ 2}_{H\ \ -H\ \ 0}\langle\rho(\mathbf{p})\rangle_{7/2\ H} \ . \tag{3.6b}$$

Combining Eqs. (3.3) and (3.6b), we get

$${}^{7/2}_{\ \ 2}\rho(\mathbf{p})=(\tfrac{8}{5})^{1/2}C^{7/2\ 2\ 7/2}_{K\ \ 0\ \ K}f_2(p)Y_{20}(\hat{\mathbf{p}}) \ , \tag{3.6c}$$

where $K=\frac{7}{2}$ for the holmium target. This gives

$${}^{7/2}_{\ \ 2}f(y)=\left[\frac{8}{5}\right]^{1/2}C^{7/2\ 2\ 7/2}_{7/2\ 0\ 7/2}[c_0\delta(y-1)+c_1\delta'(y-1)+c_2\delta''(y-1)\cdots] \ , \tag{3.7a}$$

where

$$c_n=\frac{(-1)^n}{n!}\int d^3p\left[\frac{p\cos\theta+p^2/2m^2}{m}\right]^n f_2(p)Y_{20}(\hat{\mathbf{p}}) \ . \tag{3.7b}$$

It can be seen that $c_0=c_1=0$ because of orthogonality of the spherical harmonics. Summing over all single-nucleon states yields

$$c_2=\frac{1}{3}\sum\sum_{LL'\Lambda} a_{NL'\Lambda}a_{NL\Lambda}C^{LlL'}_{\Lambda 0\Lambda}C^{LlL}_{000}\left\langle\frac{p^2}{m^2}\right\rangle_N \ , \tag{3.8a}$$

where

$$\left\langle\frac{p^2}{m^2}\right\rangle_N=\frac{\hbar\omega}{m}\left[N+\frac{3}{2}\right] . \tag{3.8b}$$

Finally, the structure function ${}^{7/2}_{\ \ 2}F_1(x)$ is

$${}^{7/2}_{\ \ 2}F_1=\int dy\int dz\,\delta(x-yz)F_1(z)f^{7/2}_{2}(y)$$
$$=\left[\frac{8}{5}\right]^{1/2}C^{7/2\ 2\ 7/2}_{7/2\ 0\ 7/2}c_2[2F_1(x)+4xF_1'(x)+x^2F_1''(x)] \ . \tag{3.9}$$

For a numerical evaluation of the above result, the quark distributions entering $F_1(x)$ are chosen to be from Duke and Owens[8] (set 1 at $Q^2=4$ GeV^2). The Nilsson orbital coefficients $a_{L\Lambda}$ needed to determine c_2 are computed from the deformed particle solutions given by Davidson.[10] The result is shown in Fig. 8.

B. Fusion corrections to ${}^{7/2}_{\ \ 2}F_1(x)$

The surface of a deformed nucleus, such as ^{165}Ho, in the body-fixed frame can be reasonably parametrized by a quadrupole form:

$$R(\Theta)=R_0[1+\beta Y_{20}(\Theta)] \ . \tag{3.10}$$

Using standard angular momentum relations, the expectation value of $R(\Theta)$ in the laboratory frame is

$$R_H(\theta,\phi)=\langle JHK|R(\Theta)|JHK\rangle$$
$$=R_0[1+\beta_{\text{lab}}(H)Y_{20}(\theta)] \ , \tag{3.11a}$$

where

$$\beta_{\text{lab}}(H)=\beta\tfrac{8}{5}(-1)^{K+H+1}C^{J\ \ J\ 2}_{H\ -H\ 0}C^{J\ \ J\ 2}_{K\ -K\ 0}$$
$$=\beta\frac{[3H^2-J(J+1)][3K^2-J(J+1)]}{J(J+1)(2J+3)(2J-1)} \ . \tag{3.11b}$$

For ^{165}Ho, $J=K=\frac{7}{2}$. The nuclear density is taken to be uniform in the body-fixed frame. In the laboratory frame:

$$\rho_H(r,\theta)=\rho_0\Theta(R_H(\theta)-r) \ . \tag{3.12}$$

Inserting this in Eq. (1.4) and ignoring terms of order β^2, th shadowing parameter K^{JH} works out to be

$$K^{JH}=K_0\left[1+\left[\frac{5}{4\pi}\right]^{1/2}\beta_{\text{lab}}(H)\right] , \tag{3.13}$$

where $K_0=(g^2/Q^2)2\pi R_0^4\rho_0^2$. For a numerical estimate we take $R_0=6.2$ fm and $\rho_0=0.165$ fm^{-3}.

The shadowing correction to the quark distribution

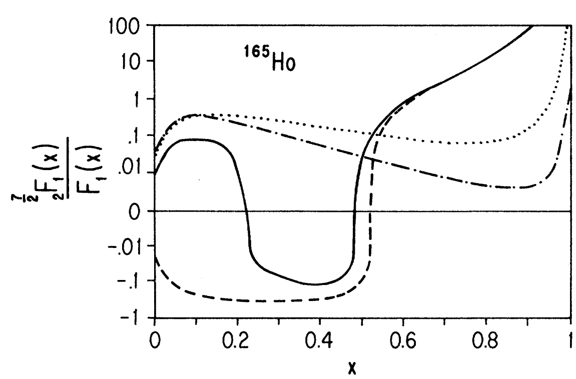

FIG. 8. (i) ${}^{7/2}_{\ \ 2}F_1(x)/F_1(x)$ in the convolution model for ^{165}Ho (dashed curve). (ii) Parton fusion correction (dotted curve) with no cutoff, i.e., $\langle z^2\rangle^{1/2}=0$—partons extend over the entire length of the nucleus. (iii) Parton fusion correction (dot-dashed curve) with $\langle z^2\rangle^{1/2}=2.4$ fm. (iv) Sum of convolution plus fusion correction with $\langle z^2\rangle^{1/2}=0$ (solid curve).

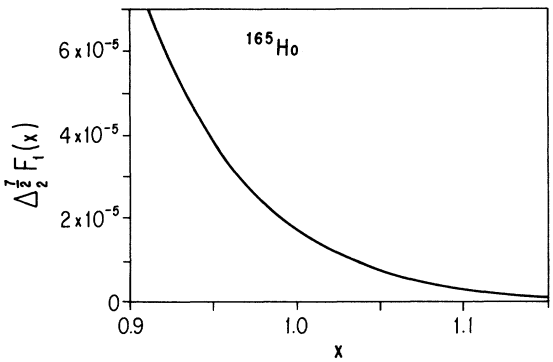

FIG. 9. $\Delta\, {}^{7/2}_{\;2}F_1(x)$ for ^{165}Ho and x beyond 1.

function $q^{JH}=q^{JH}_{\uparrow}+q^{JH}_{\downarrow}$, is obtained by combining Eqs. (1.7) and (3.13):

$$\Delta q^{JH}=K_0\left[1+\left[\frac{5}{4\pi}\right]^{1/2}\beta_{\rm lab}(H)\right]\Delta q\ . \tag{3.14}$$

Δq is the parton fusion modification of the unpolarized quark density with an implicit sum over flavor. Therefore, the correction to ${}^{7/2}_{\;2}F_1(x)$, using Eq. (1.9) is

$$\Delta\,{}^{7/2}_{\;2}F_1(x)=\tfrac{1}{2}\sum_{H=-7/2}^{7/2}(-1)^{7/2\,-H}C^{7/2}_{H}\ {}^{7/2}_{-H}\,{}^{2}_{0}\Delta q^{7/2\,H}(x)$$
$$=\left[\frac{4}{5\pi}\right]^{1/2}K_0\beta C^{7/2}_{K}\ {}^{7/2}_{-K}\,{}^{2}_{0}\Delta q(x)\ , \tag{3.15}$$

where $K=\frac{7}{2}$.

C. Results

The convolution-model prediction [Eq. (3.9)] and fusion corrections [Eq. (3.15)] to ${}^{7/2}_{\;2}F_1(x)$ are graphed in Fig. 8. The convolution-model result ${}^{7/2}_{\;2}F_1(x)$ is small and negative for $x\lesssim 0.5$. It then goes through zero and rises steeply beyond $x\approx 0.5$. The fusion corrections to ${}^{7/2}_{\;2}F_1(x)$ alter this behavior in the $x<0.4$ region where they are relatively large and positive. ${}^{7/2}_{\;2}F_1(x)$ then becomes positive for $x\lesssim 0.2$ as well. It therefore goes through zero twice. It can be seen that the fusion corrections are not very sensitive to the leakage function $W(x)$. As in the case of lithium, parton fusion extends ${}^{7/2}_{\;2}F_1(x)$ beyond $x=1$ (Fig. 9).

IV. DISCUSSION

In this paper we have presented numerical estimates for the expected convolution-model predictions for ${}^{J}_{2}F_1(x)$ for two different types of nuclei, as well as estimates based upon the assumption that the parton recombination model of Close, Qiu, and Roberts[7] is of reasonable validity. We find that for $x<0.4$ the simple additivity of quark distributions may be insufficient, and fusion effects might play a dominant role there. The possibility of polarizing nuclei for deep-inelastic-scattering experiments makes measurement of polarized structure functions such as ${}^{J}_{2}F_1(x)$ of considerable interest. We remark, however, that a weakness of the model of Ref. 7 remains a weakness of our calculations as well—a starting value of Q_0^2 in the range 4–5 GeV^2 must be assumed as in Ref. 7 and does not appear to be derivable from more fundamental principles.

We now briefly discuss how ${}^{J}_{2}F_1(x)$ can be experimentally measured: Recall that the spin-averaged cross section for deep-inelastic lepton scattering from any target is extracted using

$$\frac{d\sigma}{dx\,dy}=\frac{e^2ME}{2\pi Q^4}[1+(1-y)^2]F_1(x)\ , \tag{4.1}$$

where $F_1(x)$ is the target's spin-averaged structure function. For the lithium target ($J=\frac{3}{2}$), ${}^{3/2}_{\;2}F_1(x)$ can be extracted by polarizing the target along the beam such that the target helicity is $H=\pm\frac{3}{2}$. The cross section in this case is[2]

$$\frac{d\sigma^{3/2\ 3/2}}{dx\,dy}=\frac{e^4ME}{2\pi Q^4}[1+(1-y)^2]\left[\frac{1}{2}\,{}^{3/2}_{\;2}F_1(x)+F_1(x)\right]\ . \tag{4.2}$$

Combining Eq. (4.2) with the unpolarized measurement [see Eq. (4.1)] yields ${}^{3/2}_{\;2}F_1(x)$.

We can measure ${}^{7/2}_{\;2}F_1(x)$ for a holmium target in a similar manner. In the approximation where the holmium nucleus is assumed to have only quadrupolar deformation, the multiple structure functions ${}^{L}_{L}F_1(x)$ for $L>2$ vanish. The cross section for $J=H=\frac{7}{2}$ is therefore[2]

$$\frac{d\sigma^{7/2\ 7/2}}{dx\,dy}=\frac{e^4ME}{2\pi Q^4}[1+(1-y)^2]\left[\left[\frac{7}{24}\right]^{1/2}{}^{7/2}_{\;2}F_1+F_1\right]\ . \tag{4.3}$$

Combining Eqs. (4.1) and (4.3) gives the desired structure function.

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APPENDIX

We want to apply Villar's[9] formalism in momentum space to determine the laboratory-frame momentum distribution of nucleons, given a many-body wave function in the body-fixed frame. Let $\psi^{J}_{KH}(p_1,\ldots,p_A)$ be the Fourier transform of $\psi^{J}_{KH}(x_1,\ldots,x_A)$:

$$\psi^J_{KH}(p_1,\ldots,p_A) \equiv \int \frac{dx_1}{(2\pi)^{3/2}} \cdots \frac{dx_A}{(2\pi)^{3/2}} e^{ip_1x_1} \cdots e^{ip_Ax_A} \psi^J_{KH}(x_1,\ldots,x_A)$$
$$= \int \frac{dx_1}{(2\pi)^{3/2}} \cdots \frac{dx_A}{(2\pi)^{3/2}} \int d\Omega\, e^{ip_1x_1} \cdots e^{ip_Ax_A} D^{\dagger J}_{KH}(\Omega)\Phi(Rx_1,\ldots,R_{x_A}) . \quad (A1)$$

Then, by changing variables of integration, $y_1 = Rx_1$, and using $p \cdot R^{-1} y = Rp \cdot y$,

$$\psi^J_{KH}(p_1,\ldots,p_A) = \int d\Omega\, D^{\dagger J}_{KH}(\Omega)\mathcal{R}(\Omega) \int \frac{dx_1}{(2\pi)^{3/2}} \cdots \frac{dx_A}{(2\pi)^{3/2}} e^{ip_1x_1} \cdots e^{ip_Ax_A} \Phi(x_1,\ldots,x_A) = P^J_{KH}\Phi(p_1,\ldots,p_A) . \quad (A2)$$

This shows that we can work equally well in coordinate or momentum space using the projection operator in Eq. (3.1). Here $\psi^J_{KH}(p_1,\ldots,p_A)$ is the laboratory-frame nuclear wave function of spin J. The quantity we wish to calculate is the laboratory-frame momentum distribution

$$\langle \rho \rangle_{JH} = \frac{\int [dp] \psi^{*J}_{KH} \hat{\rho} \psi^J_{KH}}{\int [dp] \psi^{*J}_{KH} \psi^J_{KH}} , \quad (A3)$$

where the density operator $\hat{\rho}(\mathbf{p}) = \sum_{i=1}^{A} \delta(\mathbf{p} - \mathbf{p}_i)$. Suppressing division by the norm of ψ^J_{KH} for the time being, and using Eq. (A2),

$$\langle \delta(\mathbf{p} - \mathbf{p}_i) \rangle = \int [dp] \psi^{*H}_{KH} \delta(\mathbf{p} - \mathbf{p}_i) \psi^J_{KH}$$
$$= \int d\Omega\, d\Omega' D^J_{HK}(\Omega') D^{*J}_{HK}(\Omega) \int [dp] \Phi^*(R'p) \delta(\mathbf{p} - \mathbf{p}_i) \Phi(Rp) . \quad (A4)$$

Changing integration variables from $p_1, p_2, \ldots$ to $p'_1 = R'p_1$, etc, and defining $\tilde{R} \equiv RR'^{-1}$ so that $D^J_{HK}(\Omega) = D^J_{HZ}(\Omega') D^J_{ZK}(\tilde{\Omega})$,

$$\langle \delta(\mathbf{p} - \mathbf{p}_i) \rangle = \int d\Omega\, d\Omega' D^J_{HK}(\Omega') D^{*J}_{HZ}(\Omega') D^{*J}_{ZK}(\tilde{\Omega}) \int [dp'] \Phi^*(p') \delta(\mathbf{p} - R'^{-1} p'_i) \Phi(\tilde{R}p') . \quad (A5)$$

Using

$$\delta(\mathbf{p} - R'^{-1} p_j) = 1/p^2 \delta(p - p_j) \sum_{lm} Y^*_{lm}(\hat{\mathbf{p}}) Y_{lm}(R'^{-1} p_j) \quad (A6)$$

and angular momentum relations gives

$$\langle \delta(\mathbf{p} - \mathbf{p}_i) \rangle = \frac{1}{(2J+1)} \frac{1}{p^2} \sum_{lm} C^{J\,l\,J}_{H0H} C^{J\,l\,J}_{K\,m\,Z} Y^*_{lm}(\hat{\mathbf{p}}) \int d\Omega\, D^{*J}_{ZK}(\Omega) \int [dp] \Phi^* \delta(\mathbf{p} - p_i) Y_{l0}(\hat{\mathbf{p}}_i) \mathcal{R}(\Omega) \Phi . \quad (A7)$$

$\mathcal{R}(\Omega)$ rotates arguments of Φ and has projection K along the body-fixed 3 axis, and so

$$\Phi^*_K Y_{l0} \mathcal{R}(\Omega) \Phi_K = e^{-i\alpha K} e^{-i\gamma K} \Phi^*_K Y_{l0} e^{-i\beta J_y} \Phi_K , \quad (A8)$$

and

$$D^*_{ZK} = e^{+i\alpha Z} e^{+i\gamma K} d^J_{ZK}(\beta) . \quad (A9)$$

Combining Eqs. (A7), (A8), (A9), and integrating over α yields

$$\langle \delta(p - p_i) \rangle = \frac{1}{(2J+1)p^2} \sum_l C^{J\,l\,J}_{H0H} C^{J\,l\,J}_{K0K} Y_{l0}(\hat{\mathbf{p}}) \tfrac{1}{2} \int_{-1}^{+1} d(\cos\beta) d^J_{KK}(\beta) \int [dp] \Phi^*_K \delta(p - p_i) Y_{l0}(\hat{\mathbf{p}}_i) e^{-i\beta J_y} \Phi_k . \quad (A10)$$

The normalization which had been suppressed hitherto is given by

$$\int [dp] \psi^J_{KH} \psi^J_{KH} = \frac{1}{2J+1} \frac{1}{2} d(\cos\beta) d^J_{KK}(\beta) \int [dp] \Phi^*_K e^{-i\beta J_y} \Phi_K . \quad (A11)$$

Consider now the quantity

$$M(\omega) = \int [dp] \Phi^*_K e^{-i\omega S} e^{-i\beta J_y} \Phi_K , \quad (A12a)$$

where

$$J_y = j_y(1) + j_y(2) + \cdots , \quad (A12b)$$

and

$$S = s(1) + s(2) + \cdots , \quad (A12c)$$

where

$$s(i) = \delta(p - p_i) Y_{l0}(\hat{\mathbf{p}}_i) . \quad (A12d)$$

Then

$$i\frac{\partial M}{\partial\omega}\bigg|_{\omega=0}=\int[dp]\Phi_K^* S e^{-i\beta J_y}\Phi_K \tag{A13}$$

is the desired integrand in Eqs. (A10) and (A11) ($S=1$ for the normalization integral). $M(\omega)$ can be evaluated as

$$M(\omega)=\mathrm{Det}\|M_{\lambda\mu}\|=e^{\mathrm{tr}\ln[M(\omega)]}\ , \tag{A14a}$$

where

$$\begin{aligned}M_{\lambda\mu}&=\int dp\ \phi_\lambda^* e^{-i\omega s}e^{-i\beta j_y}\Phi_\mu\\&=\delta_{\lambda\mu}-i\beta j_{\lambda\mu}-\tfrac{1}{2}\beta^2 j_{\lambda\mu}^2-i\omega s_{\lambda\mu}+\cdots\\&=(1+D)_{\lambda\mu}\ .\end{aligned} \tag{A14b}$$

(Only small values of β contribute if j^2 is large.[9]) Finally, we get

$$M(\omega)=e^{-i\omega\langle S\rangle}e^{-\beta^2\langle J_y^2\rangle/2}\ , \tag{A15}$$

$$i\frac{\partial M}{\partial\omega}\bigg|_{\omega=0}=\langle S\rangle e^{-\beta^2\langle J_y^2\rangle/2}\ . \tag{A16}$$

Combining Eqs. (A10), (A11), (A13), and (A16), we obtain the required momentum distribution of nucleons:

$$\langle\rho(\mathbf{p})\rangle_{JH}=\sum_l C_{H0H}^{JlJ}C_{K0K}^{JlJ}f_l(p)Y_{l0}(\hat{\mathbf{p}})\ , \tag{A17a}$$

where

$$f_l(p)=\sum\int d\hat{\mathbf{p}}'\phi^*(p,\hat{\mathbf{p}}')Y_{l0}(\hat{\mathbf{p}}')\phi(p,\hat{\mathbf{p}}')\ . \tag{A17b}$$

The summation in Eq. (A17b) extends over all single-particle states.

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