# Twist-four corrections to deep-inelastic lepton scattering from a polarized spin-one target

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Twist-four contributions to the lowest moment of the quadrupole structure function $b_1(x)$, which is measurable in the deep-inelastic scattering of unpolarized electrons from a polarized spin-one target, are calculated in a quark-exchange model of the deuteron. The effect is small since it is proportional to the extent to which the nucleons overlap and represents a 5% correction to the twist-two result at the values of $Q^2$ of interest.

The deep-inelastic quadrupole structure function $b_1(x,Q^2)$, which exists for all hadron targets with spin $J\geq 1$, is an object of considerable interest since it provides a clear measure of exotic effects in nuclei—i.e., the extent to which the nuclear ground state deviates from being a composite of nucleons only.[1] An analysis, based upon the operator-product expansion at the twist-two level, shows that if a nucleus is simply $A$ nucleons then $b_1$ is zero except for a small and calculable effect coming from the Fermi motion of nucleons. Experimental limits on the size of $b_1$ are expected as one consequence of experiments at CERN and the DESY $ep$ collider HERA which seek to measure the spin-dependent structure functions of the proton and neutron.[2] The technique uses internal polarized gas-jet targets, thereby allowing for the use of nuclei with spins $J\geq 1$ as well.

The expected suppression of $b_1(x,Q^2)$ at the twist-two level naturally leads to the question of whether higher-twist effects—which are usually ignored in the deep-inelastic processes because they go down as powers of $1/Q^2$ relative to the leading-twist contribution—could be important. This possibility is enhanced because the general form of twist-four operators allow richer helicity structure than twist two. Generic twist-four operators include, for example, four-quark operators $\bar\psi\Gamma\psi\bar\psi\Gamma\psi$, which can transfer two units of helicity to the target. If the two active quarks come from different nucleons in a nucleus, then one has a specific nuclear mechanism for an enhancement in the twist-four contribution to $b_1(x,Q^2)$. In this Brief Report we estimate the twist-four correction to the lowest moment of $b_1(x,Q^2)$ and find that the twist-two result is not significantly modified. A measurement of $b_1(x,Q^2)$ in excess of the small amount attributable to Fermi motion would then be a clear signal for exotic nuclear components.

The framework for our calculation is provided by the operator-product expansion[3,4] of the time-ordered product of two electromagnetic currents:

$$-i\int d^4x\, e^{iq\cdot x}T(J_\mu(x)J_\nu(0))=X_{\mu\nu}+Y_{\mu\nu}\ . \tag{1}$$

The spin-two, twist-four contribution to $Y_{\mu\nu}$ is

$$Y_{\mu\nu}^{T=4,n=2}=-\frac{4g^2}{q^4}\left[g_\mu^{\mu_1}g_\nu^{\mu_2}+g_{\mu\nu}\frac{q^{\mu_1}q^{\mu_2}}{q^2}\right]O_{\mu_1\mu_2}(0)\ ,$$
$$O_{\mu_1\mu_2}(x)=\bar\psi(x)\gamma_{\mu_1}\gamma_5QT^a\psi(x)\bar\psi(x)\gamma_{\mu_2}\gamma_5QT^a\psi(x)-\text{trace}\ . \tag{2}$$

Here $Q$ is the flavor charge matrix and $T^a$, $a=1,\ldots,8$, are color-SU(3) generators normalized to $\mathrm{Tr}T^aT^b=\frac{1}{2}\delta^{ab}$. The complete expressions for $X$ and $Y$ for any spin $n$ can be found in Ref. 4. As will be explained later, $X^{T=4,n=2}$ does not contribute to the matrix element of interest in the first approximation and is therefore not displayed here.

Deep-inelastic lepton scattering is completely determined by the target matrix elements of the operators in Eq. (1). For a spin-one target polarized with spin projection $H$ along the $z$ direction, the most general $P$- and $T$-conserving, traceless and symmetric form for the forward matrix element of $O_{\mu_1\mu_2}$ is

$$\langle PH|O_{\mu_1\mu_2}|PH\rangle=A(P_{\mu_1}P_{\mu_2}-\tfrac{1}{4}M^2g_{\mu_1\mu_2})+B\Theta_{\mu_1\mu_2}^{HH}\ . \tag{3}$$

Here $\Theta_{\mu_1\mu_2}$ is a tensor constructed from the hadron's polarization vector $E_\mu(H)$,

$$\Theta_{\mu_1\mu_2}^{HH}=\frac{E_{\mu_1}^*E_{\mu_2}+E_{\mu_2}^*E_{\mu_1}}{2}-\tfrac{1}{3}(P_{\mu_1}P_{\mu_2}-g_{\mu_1\mu_2}M^2)\ . \tag{4}$$

As usual, $E^\mu(\pm1)=\mp(M/\sqrt{2})(0,1,\pm i,0)$ and $E^\mu(0)=(0,0,0,M)$ for a target at rest in the laboratory with $P^\mu=(M,0,0,0)$. The trace in (2) is obviously a rotational scalar. Hence, the reduced matrix element $B$ is conveniently obtained by taking the difference between states with $H=0$ and 1, respectively, and $\mu_1=\mu_2=3$:

$$B=-\frac{1}{M^2}(\langle PH=1|O_{33}|PH=1\rangle-\langle PH=0|O_{33}|PH=0\rangle)\ . \tag{5}$$

 

The hadron state is assumed to have the conventional normalization

$$\langle P|P'\rangle = 2E(2\pi)^3\delta^3(\mathbf{P}-\mathbf{P}') \ .$$

From the fact that $O_{\mu_1\mu_2}(x)$ is a four-quark *local* operator, it is rather obvious from (5) that $B$ must vanish if the spin-one hadron is a simple product of two nonoverlapping nucleon states at rest, or if color is not exchanged between the nucleons. (If they are in relative motion then there is a Fermi-motion correction which can be calculated as for the case of twist-two in Ref. 1. However, the Fermi-motion correction to this twist-four correction is totally negligible.) Hence, $B$ essentially measures the correlation of quarks belonging to different nucleons and, as such, is an interesting quantity. In order to estimate $B$, we shall first make an assumption that the quarks in the target are nonrelativistic. While this is hardly realistic for light quarks, it is good enough for a first estimate and enables us to discard the lower components of the quark field operators so that a reduced form for $O_{33}$ can be written

$$O_{33} = \sum_{\substack{m_1 m_1' \\ m_2 m_2'}} q^\dagger_{m_1'}\sigma^3_{m_1' m_1} Q T^a q_{m_1} q^\dagger_{m_2'}\sigma^3_{m_2' m_2} Q T^a q_{m_2} \ . \quad (6)$$

At this point we remark that the various operators which enter into $X^{T=4,n=2}_{\mu\nu}$, as given in Refs. 3 and 4, have been neglected in evaluating $B$ because they either (a) have no $\gamma_5$ (in contrast with $Y_{\mu\nu}$, which does) and so reduce to spin-independent operators in the nonrelativistic limit, or (b) contain gluon operators which are suppressed by a numerical coefficient of $\frac{1}{10}$ (and are hard to estimate). This simplifies the calculation considerably.

To estimate the matrix element for the operator $O_{33}$ above, a model of the nucleus at the quark level is needed. A simple, yet nontrivial, model for the deuteron can be constructed from two three-quark clusters, with the quarks completely antisymmetrized among themselves. Antisymmetrization has been shown to be crucial in explaining the difference in quark distributions between a nucleus and $A$ free nucleons.[5,6] The basic ansatz for the deuteron state is

$$|d\rangle = \frac{1}{\sqrt{2!}}\Phi^d_{\alpha_1\alpha_2} A^{\dagger\alpha_1} A^{\dagger\alpha_2}|0\rangle \ ,$$
$$A^{\dagger\alpha_1} = \frac{1}{\sqrt{3!}} C^{\alpha_1}_{\mu_1\mu_2\mu_3} q^\dagger_{\mu_1} q^\dagger_{\mu_2} q^\dagger_{\mu_3} \ . \quad (7)$$

In the above, $\mu$ collectively denotes a set of quark momentum, spin, isospin (or flavor), and color quantum numbers $\mu = \{\mathbf{p}, m_s, m_t, c\}$. The quark wave function $C^{\alpha_1}_{\mu_1\mu_2\mu_3}$ couples the three quarks in each cluster to the nucleon momentum, spin, and isospin, $\alpha = \{\mathbf{P}, M_s, M_t\}$. As an easily manageable ansatz, we take the momentum part of the wave function to be a product of Gaussians,

$$\exp\left[-\tfrac{1}{2}r_0^2 \sum_{i=1}^{3} (\mathbf{p}_i - \tfrac{1}{3}\mathbf{P})^2\right] \delta^{(3)}(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3-\mathbf{P}) \ , \quad (8)$$

where $r_0 = 0.8$ fm is the rms radius of the nucleon. The nucleons are then combined by the nuclear wave function $\Phi^d_{\alpha_1\alpha_2}$ to form the deuteron state. The state $|d\rangle$ differs from that usually assumed in nuclear physics only in that quarks belonging to one nucleon are also antisymmetrized with respect to quarks belonging to the second nucleon. Equivalently, one allows for the possibility of quarks being exchanged between two nucleons when they overlap to a non-negligible extent.

The matrix element of the four-quark operator $O_{33}$ in the state $|d\rangle$ specified above can be worked out[7] with the help of the anticommutation relation $\{q_\mu, q^\dagger_\nu\} = \delta_{\mu\nu}$. Basically, one must evaluate the momentum, spin-isospin, and color sums coming from the five different topologies of graphs depicted in Fig. 1. These are not Feynman graphs, they merely pictorialize the different combinations which occur when taking the matrix element $\langle d|q^\dagger q q^\dagger q|d\rangle$ and allow one to see more conveniently the momentum, spin, charge, and color flow in the system.

In calculating $B$ for the deuteron it is adequate to consider the $s$ state alone both because the $d$ state has a probability of a few percent only, and because the $l=2$ centrifugal barrier decreases the probability of nucleons overlapping. (Basically for reasons of rotational invariance, the $s$ state does not contribute to $B$ at the twist-two level whether or not there is an exchange of quarks between nucleons.) Using the Reid soft-core wave function,[8] and another which is a sum of Gaussians,[9] we have calculated the momentum-space integrals implied in Figs. 1(a)–1(e). The various momentum, spin, isospin, and color matrix elements are listed in Table I. The values calculated for $B$ are 43.4 and 16.1 MeV$^2$, respectively, for the two wave functions used. The discrepancy between these two values reflects the theoretical uncertainty regarding the overlap region of the deuteron wave function. $B$ is not sensitive to the nucleon radius $r_0$ for $r_0$ near 0.8 fm—increasing $r_0$ increases the extent of overlap between nucleons, but it also decreases the probability of finding two

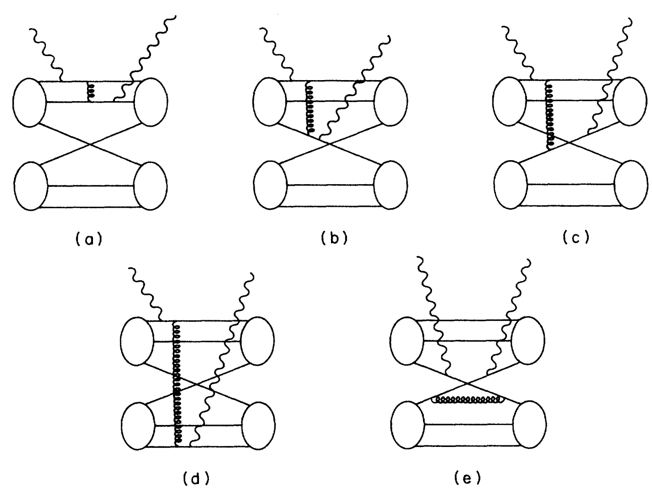


FIG. 1. Quark-exchange graphs which contribute to the matrix elements of $O_{33}$.

TABLE I. Matrix elements of the color, spin-isospin, and momentum operators corresponding to Figs. 1(a)–1(e). Weight corresponds to the frequency of occurrence of the diagrams. The first entry of the last column is the momentum matrix element for the Reid soft-core wave function (Ref. 8) and the second is for the sum of Gaussian wave functions in Ref. 9.

| | Weight | Color | Spin/isospin | Mom ($MeV^2$) |
|---|---|---|---|---|
| 1(a) | −36 | $-\frac{2}{9}$ | $-\frac{8}{243}$ | 133.8/24.7 |
| 1(b) | −72 | $-\frac{2}{9}$ | $\frac{4}{243}$ | 77.8/12.8 |
| 1(c) | −72 | $-\frac{2}{9}$ | $\frac{4}{243}$ | 77.8/12.8 |
| 1(d) | −72 | $\frac{1}{9}$ | $-\frac{2}{243}$ | 35.4/13.1 |
| 1(e) | −18 | $\frac{4}{9}$ | $\frac{20}{243}$ | 78.2/16.1 |

quarks in a fixed volume element.

We shall now relate $B$ to the structure function $b_1(x,Q^2)$, which is most easily understood in terms of forward Compton-scattering helicity amplitudes:

$$b_1(x,Q^2)=-\tfrac{1}{2}(A_{++,++}+A_{+-,+-}-2A_{+0,+0}) , \qquad (9)$$

where $A_{hH,h'H'}=\epsilon^{*\mu}(h')W^{H'H}_{\mu\nu}\epsilon^{\nu}(h)$, and $W^{H'H}_{\mu\nu}$ equals $(1/2\pi)$ times the hadron matrix element of the imaginary part of the time-ordered product in Eq. (1). Standard dispersion theory analysis then yields the twist-four correction to the lowest Nachtmann moment of $b_1$:

$$\int_0^1 dx\, x b_1(x,Q^2)= \cdots + \frac{\pi\alpha(Q^2)}{Q^2}B , \qquad (10)$$

where $\cdots$ denotes omitted twist-two and target-mass corrections. Since $\pi\alpha(Q^2)\sim 1$, the right-hand side is of order 40 $MeV^2/Q^2$ which, for $Q=1$ GeV, is $4\times10^{-5}$. This is considerably smaller than the result of $-6\times10^{-4}$ obtained from the convolution model formulas of Ref. 1. Of course, the twist-two result is itself small and $b_1(x,Q^2)$ remains a window through which exotic nuclear components could be observed.

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[1]P. Hoodbhoy, R. L. Jaffe, and A. Manohar, Nucl. Phys. **B212**, 571 (1989).
[2]HERMES Collaboration (unpublished); SMC Collaboration (private communication).
[3]R. L. Jaffe and M. Soldate, Phys. Lett. **105B**, 467 (1981).
[4]R. L. Jaffe and M. Soldate, Phys. Rev. D **26**, 49 (1982).
[5]P. Hoodbhoy and R. L. Jaffe, Phys. Rev. D **35**, 113 (1987).
[6]Arifuzzaman, S. H. Hasan, and P. Hoodbhoy, Phys. Rev. C **38**, 498 (1988).
[7]Arifuzzaman, P. Hoodbhoy, and S. Mahmood, Nucl. Phys. **A480**, 569 (1988).
[8]R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).
[9]K. Maltman and N. Isgur, Phys. Rev. D **29**, 952 (1984).