Twist-four distributions in a transversely polarized nucleon and the Drell-Yan process

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(Received 2 June 1993)

The twist-four quark and gluon distributions in a transversely polarized nucleon are identified and relations among them are discussed by using QCD equations of motion. At the order of $O(1/Q^2)$, the Drell-Yan cross section in transversely polarized nucleon-nucleon collisions is expressed in terms of these distributions.

PACS number(s): 13.85.Qk, 13.88.+e, 14.20.Dh

I. INTRODUCTION

There has been considerable interest recently in the polarization-dependent quark distributions of a transversely polarized nucleon [1–4]. These distributions, which are on the same footing as those of an unpolarized or longitudinally polarized nucleon, are just as important for characterizing the nucleon's high-energy structure. Being universal quantities, meaning that they describe the nucleon rather than a particular process, these distributions can be measured in a variety of experiments involving large momentum transfers. Although at present no parton distribution can be calculated *ab initio* from QCD, nonetheless this may eventually become possible.

A very interesting distribution $h_1(x)$, called the transversity distribution in Ref. [2], was first identified by Ralston and Soper. That this involves a helicity flip of the participating quark was recognized by Artu and Mekhfi [1]. The $h_1(x)$ is a twist-two quantity, meaning that it enters into the cross section of a hard process, such as Drell-Yan lepton pair production, unsuppressed by inverse powers of the hard momentum Q. It has been proposed that $h_1(x)$ be measured using transversely polarized beams at the BNL Relativistic Heavy Ion Collider (RHIC) [4]. The $O(1/Q^2)$, or twist-four, corrections to this process involve parton distributions with four partons. The purpose of this paper is to identify and classify these distributions and to express the Drell-Yan cross section in terms of them.

Higher twist corrections, as is well known, are notoriously complicated. Nonetheless, if one is to proceed beyond the naive parton model, it is necessary to identify all parton distributions which enter at higher twist, to study their symmetry properties, and to calculate their contributions to different hard processes. In contrast with simple distributions involving only two quark or gluon fields which occur at leading twist, higher twist distributions involve the matrix elements of the multiquark and gluon field operators at an equal light-cone time. A complete catalogue of unpolarized and longitudinally polarized distributions up to twist four, and transverse distributions up to twist three, can be found in Ref. [3].

The results of this paper, which deals exclusively with the twist-four distributions of a transversely polarized nucleon, are summarized below.

Parton distributions are introduced through parton-hadron vertices. As shown in Fig. 1, the twist-four distributions in the light-cone gauge are contained in the vertices up to four partons. Here the four-gluon vertex is absent because it cannot mix with the *chiral-odd* vertices shown. The two-quark vertex, shown in Fig. 1(a) provides one twist-four distribution $h_3(x)$. There are four distributions, named $d_i(x,y)$ with $i = 1,2,3,4$, associated with the two-quark–one-gluon vertex in Fig. 1(b). The two-quark–two-gluon vertex, Fig. 1(c), gives rise

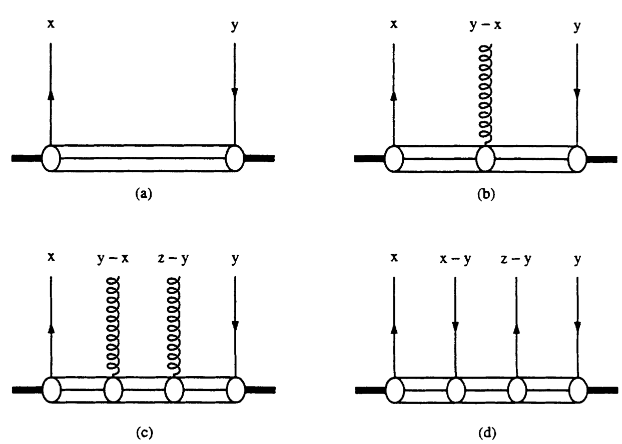

FIG. 1. Parton density matrices which can contribute to a hard process at the twist-four level.

to three distributions named herein as $D_i(x,y,z)$ with $i = 1,2,3$. Finally, the four-quark vertex [Fig. 1(d)] has two associated distributions $W_i(x,y,z)$ with $i = 1,2$. The tensor structure which gives rise to each distribution is given in the text, together with restrictions obtained from the requirement of PCT invariance. The QCD equations of motion on the light cone impose further restrictions by specifying relations between distributions involving more light-cone momentum fractions and distributions involving fewer. Including the distributions identified in Ref. [3], i.e., $h_1(x)$, $g_T(x)$, $G_1(x,y)$, and $G_2(x,y)$, a complete set is now available to deal with any hard process involving transversely polarized nucleons up to and including twist four. As one application, we have obtained an expression for the twist-four part of the Drell-Yan cross section with transversely polarized nucleons. This could provide a framework to analyze corrections if an experiment to measure $h_1(x)$ is actually performed [4].

II. TWIST-FOUR DISTRIBUTIONS IN A TRANSVERSELY POLARIZED NUCLEON

In this section, we consider the polarization-dependent twist-four distributions in a transversely polarized nucleon. To establish notation, we consider a nucleon moving in the z direction with its spin vector in the x-y plane with momentum $P^\mu = p^\mu + \frac{1}{2}M^2 n^\mu$ and $S_\perp^\mu = (0, \mathbf{S}_\perp, 0)$, where $\mathbf{S}_\perp \cdot \mathbf{S}_\perp = 1$ and p^μ and n^μ are null vectors satisfying $p^2 = n^2 = 0$ and $p \cdot n = 1$. [We sometimes use light-cone coordinates so the four components of a vector x^μ are x^+, x^-, and $x_\perp$, where $x^\pm = (1/\sqrt{2})(x^0 \pm x^3)$ and $x_\perp = (x^1, x^2)$.] A distribution of partons in such a nucleon is defined by the matrix element of quark and gluon field operators at an equal light-cone time. In our calculation, we choose the light-cone gauge $A \cdot n = 0$; thus, a gauge link is not needed in the distributions to ensure their gauge invariance. Simple dimensional reasoning enables determination of the number of inverse powers of the hard momentum which accompany a given distribution, and thus its twist. Application of PCT rules out a large number of otherwise possible structures. For further details the reader is referred to Refs. [2] and [3].

At the level of twist four, the two-quark vertex in Fig. 1(a) contains only one transverse distribution:

$$h_3(x) = \frac{1}{\Lambda^2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp | \bar{\psi}(0) \gamma_5 \, \not{S}_\perp \, \not{p} \psi(\lambda n) | PS_\perp \rangle, \tag{1}$$

where Λ is a soft scale. It can also be projected out from the quark density matrix:

$$\begin{aligned} M_{\alpha\beta}(x) &= \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp | \bar{\psi}_\beta(0) \psi_\alpha(\lambda n) | PS_\perp \rangle \\ &= \frac{1}{4}\Lambda^2 (\not{n}\gamma_5 \, \not{S}_\perp)_{\alpha\beta} h_3(x) + \cdots, \end{aligned} \tag{2}$$

where α and β label Dirac indices and dots represent other distributions which are not our concern here.

The two-quark–one-gluon vertex, Fig. 1(b), is more complex and involves two light-cone fractions since the quark and gluon can be removed from the hadron from different light-cone "positions" $x^- = \lambda n^-$ or $y^- = \mu n^-$. There are four PCT allowed transverse distributions:

$$\begin{aligned} d_1(x,y) &= \int [d\lambda\, d\mu] \langle PS_\perp | \bar{\psi}(0) iD_\perp(\mu n) \cdot S_\perp \gamma_5 \psi(\lambda n) | PS_\perp \rangle, \\ d_2(x,y) &= \int [d\lambda\, d\mu] \langle PS_\perp | \bar{\psi}(0) iD_\perp(\mu n) \cdot S_\perp \gamma_5 \tfrac{1}{2}(\not{p}\,\not{n} - \not{n}\,\not{p}) \psi(\lambda n) | PS_\perp \rangle, \\ d_3(x,y) &= \int [d\lambda\, d\mu] \langle PS_\perp | \bar{\psi}(0) iD_\perp(\mu n) \cdot iT_\perp \psi(\lambda n) | PS_\perp \rangle, \\ d_4(x,y) &= \int [d\lambda\, d\mu] \langle PS_\perp | \bar{\psi}(0) iD_\perp(\mu n) \cdot iT_\perp \tfrac{1}{2}(\not{p}\,\not{n} - \not{n}\,\not{p}) \psi(\lambda n) | PS_\perp \rangle, \end{aligned} \tag{3}$$

where $T_\perp^\alpha = \epsilon^{\alpha\beta\gamma\delta} p_\beta n_\gamma S_{\perp\delta}$ is a vector orthogonal to $S_\perp^\alpha$ with $T_\perp \cdot T_\perp = -1$ and

$$[d\lambda\, d\mu] = \frac{1}{2\Lambda^2} \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{i\lambda x} e^{i\mu(y-x)}. \tag{4}$$

$D_\perp$ is the transverse part of the covariant derivative $D^\alpha = \partial^\alpha + igA^\alpha = p^\alpha n \cdot D + n^\alpha p \cdot D + D_\perp^\alpha$. The $d_i(x,y)$ may be readily projected out of the quark-gluon density matrix:

$$\begin{aligned} M_{\mu\nu}^\alpha(x,y) &= 2\Lambda^2 \int [d\lambda\, d\mu] \langle PS_\perp | \bar{\psi}_\nu(0) iD_\perp^\alpha(\mu n) \psi_\mu(\lambda n) | PS_\perp \rangle \\ &= -\tfrac{1}{2}\Lambda^2 [S_\perp^\alpha \gamma_5 d_1(x,y) + S_\perp^\alpha \tfrac{1}{2}(\not{p}\,\not{n} - \not{n}\,\not{p})\gamma_5 d_2(x,y) - iT_\perp^\alpha d_3(x,y) - iT_\perp^\alpha \tfrac{1}{2}(\not{p}\,\not{n} - \not{n}\,\not{p}) d_4(x,y)]_{\mu\nu} + \cdots, \end{aligned} \tag{5}$$

where α is a vector index and μ, ν are spinor indices. It can be easily shown that the d_i's are real and obey the symmetry relations

$$d_1(x,y) = -d_1(y,x), \quad d_2(x,y) = d_2(y,x),$$
$$d_3(x,y) = -d_3(y,x), \quad d_4(x,y) = d_4(y,x). \tag{6}$$

Note that the index α in Eq. (5) refers to transverse polarizations ($\alpha = 1,2$) only. In the light-cone gauge, $A \cdot n = A^+ \cdot n^- = 0$, A^- is related to $A_\perp$ through QCD equations of motion. The A^- contribution in Eq. (5) gives distributions of twist higher than four.

The two-quark and two-gluon vertex, Fig. 1(c), also with transverse gluons only, has three light-cone fractions. There are three allowed distributions:

$$\begin{aligned}
D_1(x,y,z) &= \int [d\lambda\, d\mu\, d\nu] \langle PS_\perp|\bar\psi(0) iD_\perp(\nu n)\cdot iD_\perp(\mu n) \not{n}\gamma_5 \not{S}_\perp \psi(\lambda n)|PS_\perp\rangle, \\
D_2(x,y,z) &= \int [d\lambda\, d\mu\, d\nu] \langle PS_\perp|\bar\psi(0)[iD_\perp(\nu n)\cdot S_\perp i \not{D}_\perp(\mu n) + i\not{D}_\perp(\nu n) iD_\perp(\mu n)\cdot S_\perp \\
&\quad -iD_\perp(\nu n)\cdot iD_\perp(\mu n) \not{S}_\perp] \not{n}\gamma_5 \psi(\lambda n)|PS_\perp\rangle, \\
D_3(x,y,z) &= \int [d\lambda\, d\mu\, d\nu] \langle PS_\perp|\bar\psi(0) i\epsilon^{\alpha\beta\gamma\delta} p_\gamma n_\delta iD_{\perp\alpha}(\nu n) iD_{\perp\beta}(\mu n) \not{n} \not{S}_\perp \psi(\lambda n)|PS_\perp\rangle,
\end{aligned} \tag{7}$$

where

$$[d\lambda\, d\mu\, d\nu] = \frac{1}{2\Lambda^2}\frac{d\lambda}{2\pi}\frac{d\nu}{2\pi}\frac{d\mu}{2\pi} e^{i\lambda x} e^{i\mu(y-x)} e^{i\nu(z-y)}. \tag{8}$$

The above distributions can be projected out of the $qGGq$ density matrix:

$$\begin{aligned}
M^{\alpha\beta}_{\mu\nu}(x,y,z) &= 2\Lambda^2 \int [d\lambda\, d\mu\, d\nu] \langle PS_\perp|\bar\psi_\nu(0) iD^\alpha(\nu n) iD^\beta(\mu n)\psi_\mu(\lambda n)|PS_\perp\rangle \\
&= \tfrac{1}{4}\Lambda^2 [g_\perp^{\alpha\beta}\gamma_5 \not{S}_\perp \not{p} D_1(x,y,z) + \tfrac{1}{2}(S_\perp^\alpha\gamma^\beta + S_\perp^\beta\gamma^\alpha - g_\perp^{\alpha\beta} \not{S}_\perp) \not{p}\gamma_5 D_2(x,y,z) \\
&\quad + i\epsilon^{\alpha\beta\gamma\delta} p_\gamma n_\delta \not{S}_\perp \not{p} D_3(x,y,z)]_{\mu\nu} + \cdots .
\end{aligned} \tag{9}$$

Working from the definitions (7a)–(7c), we can establish the symmetry relations

$$\begin{aligned}
D_1(x,y,z) &= D_1(z,y,x), \\
D_3(x,y,z) &= -D_3(z,y,x).
\end{aligned} \tag{10}$$

Finally, the four-quark matrix element in Fig. 1(d) can be similarly analyzed. The twist-four transverse spin distributions are

$$\begin{aligned}
W_1(x,y,z) &= \frac{g^2}{2} \int [d\lambda\, d\mu\, d\nu \langle PS_\perp|\bar\psi(0) \not{n}\psi(\nu n)\bar\psi(\mu n) \not{n}\gamma_5 \not{S}_\perp \psi(\lambda n)|PS_\perp\rangle, \\
W_2(x,y,z) &= \frac{g^2}{2} \int [d\lambda\, d\mu\, d\nu] \langle PS_\perp|\bar\psi(0) \not{n}\gamma_5\psi(\nu n)\bar\psi(\mu n) \not{n} \not{S}_\perp \psi(\lambda n)|PS_\perp\rangle.
\end{aligned} \tag{11}$$

The four-quark density matrix, which is a matrix in two sets of Dirac indices, from which these can be projected is

$$\begin{aligned}
M_{\alpha\beta\gamma\delta} &= 2g^2\Lambda^2 \int [d\lambda\, d\mu\, d\nu] \langle PS_\perp|\bar\psi_\beta(0)\psi_\alpha(\nu n)\bar\psi_\delta(\mu n)\psi_\gamma(\lambda n)|PS_\perp\rangle \\
&= \tfrac{1}{4}\Lambda^2 [(\not{p})_{\alpha\beta}(\gamma_5 \not{S}_\perp \not{p})_{\gamma\delta} W_1(x,y,z) + (\gamma_5 \not{p})_{\alpha\beta}(\not{p} \not{S}_\perp)_{\gamma\delta} W_2(x,y,z)] + \cdots .
\end{aligned} \tag{12}$$

So far we have, using *PCT*, Lorentz invariance, and dimensional counting, identified *all* tensor structures needed to describe a transversely polarized nucleon at the twist-four level. However, we have further constraints available to us in the form of QCD equations of motion on the light cone:

$$i\frac{d}{d\lambda}\psi_-(\lambda n) = -\tfrac{1}{2} \not{n} i \not{D}_\perp \psi_+(\lambda n),$$

where

$$\psi_\pm = \tfrac{1}{2}\gamma^\mp\gamma^\pm\psi. \tag{13}$$

This can be used to establish relations between distributions with three light-cone fractions with those having two and one, etc. A particularly useful set of relations, which will be needed for establishing the electromagnetic gauge invariance of the Drell-Yan cross, is

$$\int dz\, D_2(x,y,z) = -y[d_1(x,y) + d_2(x,y) + d_3(x,y) + d_4(x,y)], \tag{14a}$$

$$\int dz\, D_2(z,x,y) = x[d_1(x,y) + d_3(x,y) - d_2(x,y) - d_4(x,y)], \tag{14b}$$

$$\int dy\, dz\, D_2(y,x,z) = x^2 h_3(x). \tag{14c}$$

To convince the reader that we have obtained a complete set of distributions, we now briefly describe the procedure that we have used to identify them. First of all, the light-cone constraint equations such as (13) allow us to express ψ_- and A^- in terms of the independent fields ψ_+ and $A_\perp$. After replacing ψ_- and A^- in Eqs. (1) and (3), one sees that all the twist-four distributions contain four independent fields. In fact, because the dimensionality of a distribution is generated from quark and gluon fields that it contains, the number of independent fields in a distribution determines its twist entirely. Thus, no parton-hadron vertices other than those in Fig. 1 contain twist-four distributions. Second, we interpret each of the vertices in Fig. 1 as a parton density matrix with a set of vector or spinor indices. We expand the density matrix in terms of various Lorentz structures containing $p, n, S_\perp$, and Dirac matrices. We systematically write down all independent Lorentz structures according to parity, charge conjugation, and time reversal. The twist-four distributions are just the coefficients of these Lorentz structures, with dimension 2 units less than that of the density matrix. [The dimensions of $p, S_\perp$, and n are 1, 0, and -1, respectively.] All the twist-four distributions effectively have a mass dimension of 2, which we scale out using $\Lambda_{\rm QCD}$, making the distributions dimensionless.

All distributions discussed so far are implicitly dependent on the renormalization scale. Their evolution through radiative processes, however, is a complicated issue and beyond the scope of this paper.

III. THE TRANSVERSELY POLARIZED DRELL-YAN PROCESS

We now consider application to the Drell-Yan production of lepton pairs in transversely polarized nucleon-nucleon collisions. Calculations beyond leading twist require considerable formal development, as in the work of Ellis, Furmanski, and Petronzio [5], and more recently by Qiu and Sterman [6], and Jaffe and Ji [2]. A collinear expansion of the parton momenta is carried out, different Feynman diagrams are combined together to arrive at color gauge invariance, etc. However, a simpler set of rules emerges from the formalism, which is summarized in Ref. [3]. These rules may be readily applied to the Drell-Yan process by first drawing the set of all diagrams which give rise to a lepton pair in the final state, and which do not contain more than four partons belonging to a single nucleon. The set of diagrams contributing up to twist four is given in Figs. 2–5. All diagrams of a given topology must be included, although only one has actually been shown in the figures.

The hadron tensor that appears in the inclusive Drell-Yan cross section can be written as

$$W^{\mu\nu} = \int dx\, dy (2\pi)^4 \delta^4(Q - xP_A - yP_B)\Omega^{\mu\nu}(x,y), \tag{15}$$

where Q is the four-momentum of the virtual photon, P_A and P_B are momenta of the nucleons A and B respectively, and x (> 0) and y (> 0) are the momentum fractions of the nucleons carried by quarks or antiquarks. For our purpose, we are interested in only the polarization-dependent part of the tensor. In the following discussion, we assume that a quark from A annihilates with an antiquark from B. The contribution from the opposite case, an antiquark from A annihilating a quark from B, is obtained by substituting $x \to -x$ and $-y \to x$.

To give an example of the application of the rules in Ref. [3], we calculate the contribution from Fig. 2(a):

$$\Omega^{\mu\nu}(x,y)|_{\rm Fig.\ 2(a)} = (-1){\rm Tr}\gamma^\nu M_A(x)\gamma^\mu M_B(-y). \tag{16}$$

The factor (-1) arises from the anticommuting nature of the fermion fields, and an implicit trace over color is understood. $M_A(x)$ and $M_B(-y)$ are the transverse-polarization-dependent part of the quark density matrix:

$$M(x) = \tfrac{1}{2}\gamma_5\, \not{S}_\perp\, \not{p} h_1(x) + \tfrac{1}{2}\Lambda\gamma_5\, \not{S}_\perp g_T(x) + \tfrac{1}{4}\Lambda^2\, \not{n}\gamma_5\, \not{S}_\perp h_3(x), \tag{17}$$

where we have neglected the Dirac indices.

Working out the fermion and color traces, we find

$$\Omega^{\mu\nu}(x,y)|_{\rm Fig.\ 2(a)} = \tfrac{1}{4}C_F\Lambda^2(S_{\perp A}\cdot S_{\perp B})[P_B^\mu P_B^\nu h_1^B(-y)h_3^A(x) + P_A^\mu P_A^\nu h_3^B(-y)h_1^A(x)]\frac{1}{P_A\cdot P_B}$$
$$- \tfrac{1}{4}C_F\Lambda^2\left[S^{\mu\nu} - S_{\perp A}\cdot S_{\perp B}\frac{P_A^\mu P_B^\nu + P_A^\nu P_B^\mu}{P_A\cdot P_B}\right] g_T^B(-y)g_T^A(x), \tag{18}$$

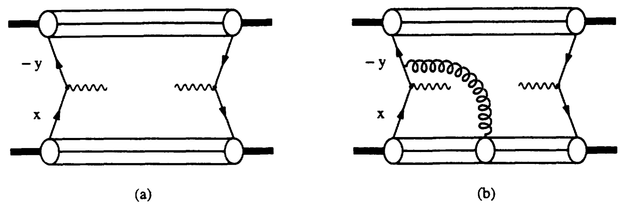

FIG. 2. A quark from hadron A (lower part) annihilates with an antiquark from hadron B (upper part) leading to a large mass virtual photon. Other diagrams of this type are obtained by allowing the gluons in (b) to interact with the quark line on the right, and interchanging A and B.

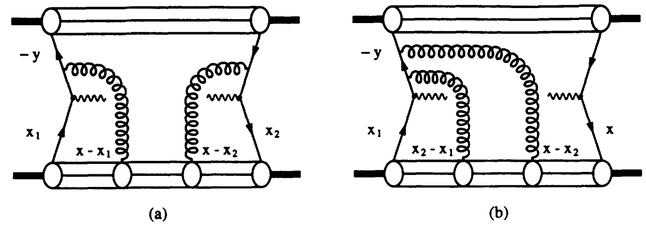

FIG. 3. Representative diagrams which give chiral-odd contributions to the twist-four Drell-Yan cross section.

where

$$S^{\mu\nu} = S^\mu_{\perp A} S^\nu_{\perp B} + S^\nu_{\perp A} S^\mu_{\perp B} - g_T^{\mu\nu} S_{\perp A}\cdot S_{\perp B}, \tag{19}$$

and S_A and S_B are the polarization vectors of the nucleons A and B, respectively. The first term contains only the chiral-odd distributions and the second the chiral-even distributions. Quark helicity conservation for massless quarks dictates that the Drell-Yan cross section consists only of the combinations of chiral-odd–chiral-odd distributions or chiral-even–chiral-even distributions. $C_F = (N^2-1)/2N = \frac{4}{3}$ is a color factor for three colors.

Figure 2(b) contains one gluon coming from the nucleon A, and the corresponding expression for $\Omega^{\mu\nu}$ is

$$\Omega^{\mu\nu}(x,y)|_{\text{Fig. 2(b)}} = (-1)\int dz\,\mathrm{Tr}\left[\gamma^\nu M_A^\alpha(z,x)\gamma^\mu M_B(-y) i\gamma_\alpha \times \frac{i}{(z-x)\not{P}_A - y\not{P}_B}\right], \tag{20}$$

where the twist-four part of the matrix $M_A(z,x)$ is given in Eq. (5), and the twist-three part in Eq. (43) in Ref. [3]. After a lengthy calculation, we find the contributions from all four diagrams having the Fig. 2(b) topology:

$$\begin{aligned}\Omega^{\mu\nu}(x,y)|_{\text{Fig. 2(b)}} = {}& -\tfrac{1}{4}C_F\Lambda^2(S_{\perp A}\cdot S_{\perp B})\frac{P_A^\mu P_B^\nu + P_A^\nu P_B^\mu}{P_A\cdot P_B}\left[\frac{h_1^B(-y)}{y}x h_3^A(x) + \frac{h_1^A(x)}{x} y h_3^B(-y)\right] \\ & +\tfrac{1}{4}C_F\Lambda^2(S_{\perp A}\cdot S_{\perp B})g_T^{\mu\nu}\left[\frac{h_1^B(-y)}{y}\int dw\frac{dz}{z}D_2^A(x,z,w) + \frac{h_1^A(x)}{x}\int dw\frac{dz}{z}D_2^B(-y,-z,-w)\right] \\ & -\tfrac{1}{2}C_F\Lambda^2(S_{\perp A}\cdot S_{\perp B})\left[\frac{x^2P_A^\mu P_A^\nu + y^2 P_B^\mu P_B^\nu}{xyP_A\cdot P_B}\right]g_T^B(-y)g_T^A(x) \\ & +\tfrac{1}{2}C_F\Lambda^2(S_{\perp A}\cdot S_{\perp B})g_T^{\mu\nu}\left[g_T^B(-y)\int dz\frac{G_1^A(x,z)-G_2^A(x,z)}{x-z}\right. \\ & \left. -g_T^A(x)\int dz\frac{G_1^B(-y,-z)-G_2^B(-y,-z)}{y-z}\right]. \end{aligned} \tag{21}$$

The first two terms in the above formula are chiral odd, and the second two are chiral even. The transverse metric tensor has only $g_T^{11} = g_T^{22} = -1$ as nonzero components.

For the diagrams with two gluons from the same nucleon shown in Figs. 3(a) and 3(b), there are only chiral-odd contributions. Using the density matrix in Eq. (9), we find

$$\Omega^{\mu\nu}(x,y)|_{\text{Fig. 3(a)}} = \tfrac{1}{4}C_F\Lambda^2(S_{\perp A}\cdot S_{\perp B})\left[x^4P_A^\mu P_A^\nu h_1^B(-y)h_3^A(x) + y^4 P_B^\mu P_B^\nu h_3^B(-y)h_1^A(x)\right]\frac{1}{x^2y^2P_A\cdot P_B}, \tag{22}$$

$$\begin{aligned}\Omega^{\mu\nu}(x,y)|_{\text{Fig. 3(b)}} = {}& -\tfrac{1}{4}C_F\Lambda^2 S^{\mu\nu}\left[\frac{h_1^B(-y)}{y}\int dx_1dx_2\frac{D_1^A(x,x_2,x_1)+D_3^A(x,x_2,x_1)}{x-x_1}\right. \\ & \left. +\frac{h_1^A(x)}{x}\int dx_1dx_2\frac{D_1^B(-y,-x_2,-x_1)+D_3^B(-y,-x_2,-x_1)}{y-x_1}\right]. \end{aligned} \tag{23}$$

In Eq. (22), we have made use of Eq. (14c). Clearly, the chiral-odd longitudinal part from Eq. (22), combined with these from Eqs. (18) and (21), is electromagnetically gauge invariant.

There are four diagrams in which one gluon comes from A and the other comes from B. These diagrams contribute only to the chiral-even part of the cross section. Our calculation shows

$$\Omega^{\mu\nu}(x,y)|_{\text{Fig. 4(a)}} = \tfrac{1}{4}C_F\Lambda^2(S_{\perp A}\cdot S_{\perp B})\frac{P_A^\mu P_B^\nu + P_A^\nu P_B^\mu}{P_A\cdot P_B}g_T^B(-y)g_T^A(x), \tag{24}$$

$$\begin{aligned}\Omega^{\mu\nu}(x,y)|_{\text{Fig 4(b)}} = {} & C_F\Lambda^2(S_{\perp A}\cdot S_{\perp B})g_T^{\mu\nu}\int\frac{dz\,dw}{z(w-y)}[G_1^A(x,w)-G_2^A(x,w)] \\ & \times[G_1^B(-z,-y)-G_2^B(-z,-y)] + (x\leftrightarrow -y, A\leftrightarrow B) \\ & -\tfrac{1}{2}C_A\Lambda^2(S_{\perp A}\cdot S_{\perp B})g_T^{\mu\nu}\int\frac{dz\,dw}{z(w-x)}[\tilde{G}_1^A(x,w)-\tilde{G}_2^A(x,w)] \\ & \times[\tilde{G}_1^B(-z,-y)-\tilde{G}_2^B(-z,-y)] + (x\leftrightarrow -y, A\leftrightarrow B),\end{aligned} \tag{25}$$

where $C_A = 3$. The second term in Eq. (25) requires some explanation. $\tilde{G}_i(x,w)$ is defined exactly as $G_i(x,w)$, except that the covariant derivative $iD^\alpha = i\partial^\alpha - gA^\alpha$ is replaced by the gluon potential $-gA^\alpha$. The transverse momentum expansion of Fig. 2(a) from which the partial derivative emerges does not produce any terms multiplied by C_A. Since the final result has to be gauge invariant, we replace the $A^\perp$ by $in^-F^{+\perp}/(w-x)$, and thus $\tilde{G}_1(x,w)$ is manifestly gauge invariant. And

$$\begin{aligned}\Omega^{\mu\nu}(x,y)|_{\text{Fig. 4(c)}} = {} & C_F\Lambda^2\mathcal{S}^{\mu\nu}\int\frac{dz}{z}\frac{dw}{w}[G_1^A(x,w)+G_2^A(x,w)][G_1^B(-y,-z)+G_2^B(-y,-z)] \\ & -\tfrac{1}{2}C_A\Lambda^2\mathcal{S}^{\mu\nu}\int\frac{dz}{z}\frac{dw}{w}[\tilde{G}_1^A(x,w)+\tilde{G}_2^A(x,w)][\tilde{G}_1^B(-y,-z)+\tilde{G}_2^B(-y,-z)],\end{aligned} \tag{26}$$

$$\Omega^{\mu\nu}(x,y)|_{\text{Fig. 4(d)}} = -C_A\Lambda^2(S_{\perp A}\cdot S_{\perp B})g_T^{\mu\nu}\int\frac{dz}{z-y}\frac{dw}{w-x}[\tilde{G}_1^A(x,w)\tilde{G}_1^B(-y,-z)-\tilde{G}_2^A(x,w)\tilde{G}_2^B(-y,-z)]. \tag{27}$$

Again, it is simple to see that the chiral-even longitudinal part in Eq. (24), combined with those from Eqs. (18) and (21), is electromagnetically gauge invariant.

Finally, we consider the four-quark diagram shown in Fig. 5. Evaluation requires a double trace as there are two quark loops. This is straightforwardly done, and using Eq. (12) we get

$$\begin{aligned}\Omega^{\mu\nu}(x,y)|_{\text{Fig. 5}} = {} & -\tfrac{1}{4}C_F\Lambda^2\mathcal{S}^{\mu\nu}\frac{h_1^B(-y)}{2y}\int\frac{dz}{z-x}\frac{dw}{w-x}\sum_{i=1}^{2}[W_i^A(x,z,w) \\ & +W_i^A(x-z,-z,w-z)-W_i^A(x,z,z-w)-W_i^A(x-z,-z,-w)] \\ & -\tfrac{1}{4}C_F\Lambda^2\mathcal{S}^{\mu\nu}\frac{h_1^A(x)}{2x}\int\frac{dz}{z-y}\frac{dw}{w-y}\sum_{i=1}^{2}[W_i^A(-y,-z,-z+w) \\ & +W_i^A(x-z,-z,w-z)-W_i^A(-y,-z,-w)-W_i^A(-y+z,z,-w+z)].\end{aligned} \tag{28}$$

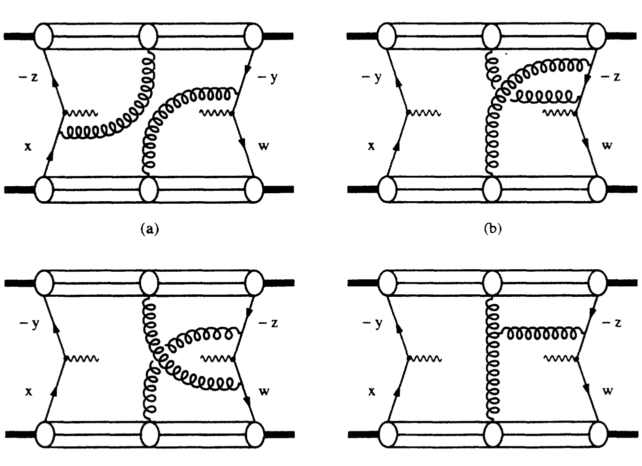

FIG. 4. Diagrams involving one gluon from each nucleon which give chiral-even contributions to the Drell-Yan cross section.

The different terms in Eq. (28) come from eight different diagrams which are obtained from Fig. 5 by changing the direction of the arrow (quark $\rightarrow$ antiquark) and attaching the twist-four distributions to a different side.

It may be readily verified that our final result for $W^{\mu\nu}$, which is given by the sum of all contributions from diagrams 2–5, is electromagnetically gauge invari-

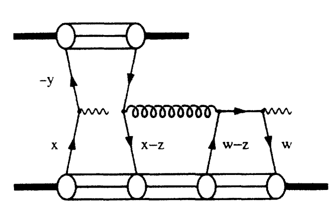

FIG. 5. Four-quark diagram contributing to the Drell-Yan twist-four cross section.

ant: $q^\mu W_{\mu\nu} = W_{\mu\nu} q^\nu = 0$. This is an important partial check on the correctness of our calculation. The result is also complete because all possible twist-four vertices have been included.

IV. SUMMARY

In this paper, we introduce the polarization-dependent twist-four distributions in a transversely polarized nucleon. The most general ones contain three light-cone momentum fractions and are D_i and W_i defined in Eqs. (7) and (11). These distributions, together with the twist-two and twist-three distributions defined for the same nucleon state, form a complete set for describing any hard scattering processes involving transverse polarization.

As an example, we have expressed the twist-four correction to the Drell-Yan process in terms of these distributions. The final result is gauge invariant in both electromagnetic and color interactions. For lack of information on these distributions, we cannot assess the numerical significance of the corrections other than a simple dimensional analysis. However, our formulas can be coupled with model calculations of the distributions to give a detailed prediction.

ACKNOWLEDGMENTS

This work was supported in part by funds provided by the U.S. Department of Energy (DOE) under Contract No. DE-AC02-76ER03069. P.H. was supported by NSF Grant No. INT-9122027.

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