Solutions to Relativity Test #1

Q.1 Apply usual length contraction formula: $L = L_0/\gamma$ so $\gamma = \frac{1}{0.8} = \frac{5}{4}$

$\gamma^2 = \frac{1}{1-\beta^2} \Rightarrow \frac{25}{16} = \frac{1}{1-\beta^2} \therefore 25 - 25\beta^2 = 16$

$\beta^2 = \frac{9}{25}$ and so $\beta = \frac{v}{c} = \frac{3}{5}$

Q.2 Without time dilation the distance travelled before decaying is $0.6c \times 2\mu s = \frac{3}{5} \times 3\times 10^8 \frac{m}{sec} \times 2.10^{-6} sec = \frac{18}{5} \times 10^2 m$.

With time dilation the particle's life is increased by amount γ,

$\gamma = \frac{1}{\sqrt{1-\left(\frac{3}{5}\right)^2}} = \frac{1}{\sqrt{1-\frac{9}{25}}} = \frac{1}{\sqrt{16/25}} = \frac{5}{4}$.

So the actual distance traveled is $\frac{5}{4} \times \frac{18}{5} \times 10^2 = 450\,m$

Q.3 Let $\Delta t = t_2 - t_1$, $\Delta x = x_2 - x_1$ be the ~~diff~~ separation in time and position between the two events. The Lorentz transformation is,

$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$. For simultaneity in S' this must be zero.

$\therefore \Delta t - \frac{v}{c^2}\Delta x = 0$. Hence $v = c^2 \frac{\Delta t}{\Delta x}$ is the speed of S' relative to S. But this must be less than c. So $c^2\frac{\Delta t}{\Delta x} < c$ or $\Delta t < \frac{\Delta x}{c}$.

Q.4 By the relativistic velocity addition formula, the observer in spaceship B will see spaceship A approaching at $v = \frac{0.8c + 0.5c}{1 + \frac{0.8c\,0.5c}{c^2}}$

$\therefore v = \frac{1.3}{1.4}c \approx 0.93c$

The frequency observed by B will be $\nu = \sqrt{\frac{1+0.93}{1-0.93}}\,\nu_0$

(you don't need to simplify, but the answer is $\nu = 5.25\nu_0$)