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MEASURING THE QCD COUPLING CONSTANT α_s INSIDE A NUCLEUS *

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ABSTRACT

Using the results of perturbative QCD for deep inelastic scattering of leptons from a polarized hadron target, we show how the QCD coupling constant α_s in a nuclear medium can be extracted in a model independent way. Predictions for α_s thus obtained are different for two models used to explain the EMC effect in nuclei, and hence the procedure can be used in principle to discriminate between the two.

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Any attempt to rigorously derive from QCD either nucleon or nuclear structure meets an impasse because the theory is intractable in the non-perturbative region. Nevertheless, there is still an interesting question that can be asked - are the strengths of the elementary vertices qqg, ggg, $gggg$ of perturbative QCD the same both inside and outside a nuclear environment? Indeed, there is no compelling reason for them to be identical because the QCD Lagrangian does not contain any mass scale; the necessity of renormalization imposes a scale into the problem which does not have to be universal.

In this note we show that this question can be resolved directly by an experiment involving the deep inelastic scattering (DIS) of polarized leptons from, successively, a polarized nucleon and polarized nuclear target. Two essential physical ideas are invoked. First, insofar as the formalism of polarized or unpolarized DIS is concerned, it is completely irrelevant whether the target is a nucleon or whether it is a nucleus with $S = \frac{1}{2}$, $I = \frac{1}{2}$ and identical z components. Many nuclei have these quantum numbers in their ground states, and mirror pairs include (p,n), $(^3He, ^3H)$, $(^{13}N, ^{13}C)$, $(^{15}O, ^{15}N)$, $(^{19}Ne, ^{19}Fe)$, $(^{29}P, ^{29}Si)$, $(^{31}S, ^{31}P)$, and so on. Of course, for heavier nuclei the electromagnetic force becomes increasingly more effective in breaking isospin symmetry. In DIS one measures various components of the tensor $W_{\mu\nu}$,

$$W_{\mu\nu} = \int d^4y \, e^{iq\cdot y} \left\langle ps \left| [J_\mu(y), J_\nu(0)] \right| ps \right\rangle . \tag{1}$$

The state $|ps\rangle$, where p is the target 4-momentum and s is the relativistic spin, is a very complex object - and even more so for a nucleus than for a nucleon. However, the usual decomposition [1)] of (1) into the structure functions F_1, F_2, g_1 and g_2 remains unchanged if one considers nuclei of the type mentioned.

The second point to be exploited is the essential similarity between the structure function $g_1(x,Q^2)$, measurable only in the DIS of polarized leptons from a polarized hadronic target, to the more familiar structure function $F_2(x,Q^2)$ which is measured in unpolarized DIS as well. F_2 is, of course, very interesting because it manifests the EMC effect [2)], i.e. the non-addivity of nucleon structure functions. Both g_1 and F_2 are weighted combinations of $q_i^{\pm}(x,Q^2)$, which is the probability of finding a quark of flavour i with helicity parallel (+) or anti-parallel (-) to the helicity of the parent spin = $\frac{1}{2}$ hadron. In Table I is shown a comparison between the polarized and unpolarized cases. Notice that the operators which enter the operator product expansion are virtually the same, the only difference arising from the dimensionless and spinless γ_5 operator which is present in the polarized case. Consequently, the evolution of the moments of both g_1 and F_2 are determined by identical anomalous dimensions d_n.

We now briefly recall some features of the treatment of deep inelastic scattering using the operator product expansion [3].

Unpolarized scattering at high Q^2 is dominated by twist two operators with matrix elements of the type

$$\langle p|\bar{\psi}\gamma^{\mu_1}D^{\mu_2}\ldots D^{\mu_n}\psi|p\rangle = A_n\, p^{\mu_1}\ldots p^{\mu_n} \quad . \tag{2}$$

Total symmetrization of indices is implicitly understood. A_n characterizes the target and cannot be computed perturbatively. Furthermore, it depends on the choice of renormalization point, $A_n = A_n(\mu^2)$. It enters into a sum rule for the moment of $F_2(x,Q^2)$

$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx\, x^{n-2}\, F_2(x,Q^2) \\ &= A_n\, C^n(Q^2,g) \quad . \end{aligned} \tag{3}$$

Here $C^n(Q^2,g)$ is a quantity which has been computed perturbatively upto order g^2 by Floratos, Ross and Sachrajda [4] (see also the appendix of Ref.5). $C^n(Q^2,g)$ has, of course, a renormalization point dependence which compensates that of A_n.

Polarized scattering at high Q^2 is also dominated by twist two operators, the sole difference relative to the previous case being that one now has a γ_5 occurring in the matrix element

$$\langle ps|\bar{\psi}\gamma_5\gamma^{\mu_1}D^{\mu_2}\ldots D^{\mu_n}\psi|ps\rangle = a_n\, s^{\mu_1}p^{\mu_2}\ldots p^{\mu_n} \quad . \tag{4}$$

Again, total symmetrization of indices is understood and now a_n characterizes the non-perturbative spin structure of the hadron. The sum rule analogous to (3) is

$$\begin{aligned} \mathcal{M}_n(Q^2) &= \int_0^1 dx\, x^{n-1}\, g_1(x,Q^2) \\ &= a_n\, E^n(Q^2,g) \quad . \end{aligned} \tag{5}$$

The Q^2 dependence in E^n comes from radiative corrections to the basic parton model and has been computed by Kodaira, Matsuda, Sasaki and Uematsu [5] upto the one loop level.

Taking the results for $C^n(Q^2,g)$ from Floratos et al. [4] and $E^n(Q^2,g)$ from Kodaira et al. [5], the ratio of $\mathcal{M}_n$ to M_n becomes

$$\frac{\mathcal{M}_n(Q^2)}{M_n(Q^2)} = \frac{a_n}{A_n}\left[1 - \frac{2}{3}\,\frac{2n+1}{n(n+1)}\,\alpha_s(Q^2) + \ldots\right] \quad . \tag{6}$$

Terms of order α_s^2 have been ignored. The simplicity of the result arises because complicated factors determining the Q^2 evolution were common both to the numerator and denominator of (6) and so cancelled in the ratio. From (6) it follows that

$$\frac{\alpha_s(Q_1^2)}{\pi} - \frac{\alpha_s(Q_2^2)}{\pi} = \frac{3n(n+1)}{2(2n+1)}\left[1 - \frac{R_n(Q_1^2)}{R_n(Q_2^2)}\right] \tag{7}$$

where

$$R_n(Q^2) = \frac{\mathcal{M}_n(Q^2)}{M_n(Q^2)} \quad . \tag{8}$$

The relation (7) is an interesting result. It states the following: first perform a deep inelastic electron or muon scattering experiment from a polarized spin = $\frac{1}{2}$ isospin = $\frac{1}{2}$ hadron at a fixed Q_1^2 and extract the n^{th} moments of the structure functions $g_1(x,Q_1^2)$ and $F_2(x,Q_1^2)$. Then repeat at a different value of four momentum transfer Q_2^2 and obtain the difference in α_s from (7), which must hold for all positive integers n. If Q_2^2 could be made very large - for arguments sake let us take $Q_2^2 = \infty$ - then by asymptotic freedom $\alpha_s(Q_2^2) = 0$ and so, putting $Q_1^2 = Q^2$,

$$\frac{\alpha_s(Q^2)}{\pi} = \frac{3n(n+1)}{2(2n+1)}\left[1 - \frac{R_n(Q^2)}{R_n(\infty)}\right] \quad . \tag{9}$$

Since the polarized hadron could be either a nucleon or a nucleus, one is directly investigating the elementary QCD vertices in different environments. In fact, (7) constitutes a non-trivial test for the theory since it must hold for all positive integers n.

At this point we remind the reader that the result (7) required the full $O(\alpha_s)$ corrections to the moments - leading log corrections would not have been sufficient. Also, it was important to have checked its invariance under a change of renormalization scheme. While this result may or may not prove useful ultimately, it is rigorous and model independent. At the very least, it allows for a theoretical experiment which addresses a key issue in nuclear physics.

Assuming that such an experiment was carried out, would one expect to see a difference between α_s thus extracted from nucleons relative to nuclei? We consider here two popular interpretations of the EMC effect. The first is the "rescaling" hypothesis of Close, Jaffe, Roberts and Ross[6] which, stated concisely, is the following: matrix elements of all twist two operators measured in DIS from nuclei are related by a simple change of Q^2 scale to corresponding matrix elements measured in DIS from free nucleons. The intuitive basis behind rescaling is that the nuclear environment changes the scale of confinement, and quarks and gluons are able to traverse relatively larger distances. In the present context, rescaling implies that in going from a proton to (say) ^3He, the n^{th} moments would change according to $M_n(Q^2) \rightarrow M_n[\xi(Q^2)\, Q^2]$ and $\mathcal{M}_n(Q^2) \rightarrow \mathcal{M}_n[\xi(Q^2)\, Q^2]$. Consequently, the value of α_s extracted from (7) or (9) will be $\alpha_s(Q^2)$ and $\alpha_s[\xi(Q^2)\, Q^2]$ respectively. The function $\xi(Q^2)$ is unity for the nucleon and greater than one for nuclei. It has been computed in Ref.6 and at $Q^2 = 20\ GeV^2$ $\xi(^3He) \approx 1.20$ and $\xi(^{13}C) \approx 1.60$. Consequently, rescaling predicts that at this value of Q^2 the measured value of α_s will shift from $\alpha_s(\text{proton}) = 0.242$ to $\alpha_s(^3He) = 0.234$ to $\alpha_s(^{13}C) = 0.222$. The value assumed for Λ_{QCD} was $\Lambda_{QCD} = 250$ MeV but there is a fair amount of uncertainty here. However, for any Λ_{QCD}, rescaling predicts that the measured vaue of α_s will decrease with increasing nuclear density. By contrast, it is easy to see that any convolution type model (with or without extra pions, isobars, or binding effects) rigorously gives zero shift. Hence, at least in principle, one can hope to distinguish between these two ways of dealing with quark degrees of freedom in nuclei.

Finally, it must be conceded that the deep inelastic scattering of leptons from a polarized spin $= \frac{1}{2}$ nucleus is a difficult experimental task. The problem is that only very thin polarized ^3He targets can be made at present, whereas at large Q^2 values the smallness of the cross-section requires thick targets. Heating of the target, with consequent depolarization, would be a serious problem. Neverthelesss there are so many uses of thick polarized nuclear targets - and the model independent analysis presented here provides one such instance - that serious thought should be given to their development by experimentalists.

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REFERENCES

1) See, for example, An Introduction to Quarks and Partons, F.E. Close (Academic Press, London 1979).

2) J.J. Aubert et al. Phys. Lett. 123B, 275 (1983). For a recent update on the theoretical status of EMC effect explanations see "Implications of QCD for nuclear physics", R.L. Jaffe, MIT CTP preprint (1987).

3) See, for example, E. Reya, Phys. Rep. 69, 195 (1981).

4) E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. B129, 66 (1977); B139, 545 (1978).

5) J. Kodaira, S. Matsuda, K. Sasaki and D. Uematsu, Nucl. Phys. B159, 99 (1979).

6) F.E. Close, R.L. Jaffe, R.G. Roberts and C.G. Ross, Phys. Rev. D31, 1004 (1985).